FORMULA SHEET FOR
M.S. COMPREHENSIVE EXAMINATION IN PHYSICAL ELECTRONICS

A. Electromagnetic Fields and Transmission Lines

Summary of Important Definitions and Results
(Notation and symbols follow the convention in *Engineering Electromagnetics*, 7th ed.,
W. H. Hyatt and J. A. Buck)

1.1 Vector Algebra

Dot Product $\mathbf{A} \cdot \mathbf{B} = |A||B| \cos \Theta_{AB} = A_x B_x + A_y B_y + A_z B_z$

Cross product $\mathbf{A} \times \mathbf{B} = |A||B| \sin \Theta_{AB} \; \mathbf{a_N}$

$$A \times B = \begin{vmatrix} a_x & a_y & a_z \\ A_x & A_y & B_z \\ B_x & B_y & B_z \end{vmatrix}$$

1.2. Vector Calculus

Divergence $\nabla . D = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}$

Gradient $\nabla V = \frac{\partial V}{\partial x}\mathbf{a}_x + \frac{\partial V}{\partial y}\mathbf{a}_y + \frac{\partial V}{\partial y}\mathbf{a}_z$

Curl

$$\nabla \times H = \left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right)\mathbf{a}_x + \left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}\right)\mathbf{a}_y + \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right)\mathbf{a}_z$$

Laplacian $\nabla^2 V = \frac{\partial^2 V}{\partial x} + \frac{\partial^2 V}{\partial y} + \frac{\partial^2 V}{\partial z}$

Vector Laplacian $\nabla^2 F = \nabla(\nabla.\mathbf{F}) - \nabla \times \nabla \times \mathbf{F}$

1.3 Integral theorems

Divergence Theorem $\oint_S \mathbf{D}.d\mathbf{S} = \int_{vol} \nabla.\mathbf{D}\, dv$

Stokes' Theorem $\oint \mathbf{H}.d\mathbf{L} = \int_S (\nabla \times \mathbf{H}).d\mathbf{S}$

2.1 Coulomb's law

$$F_2 = -F_1 = \frac{Q_1 Q_2}{4\pi\varepsilon_o R^2}\frac{\mathbf{r}_2 - \mathbf{r}_1}{|\mathbf{r}_2 - \mathbf{r}_1|} \;;$$

where $\varepsilon o = 8.854 \times 10^{-12}$ F/m = $(1/36\pi)\ 10^{-9}$ F/m

Electric Field

Definition $E = F_{test} / Q_{test}$

Due to Point Charge $E = \frac{Q}{4\pi\varepsilon_0 r^2}\mathbf{a}_r$

Due to Volume distribution $E(r) = \int_{vol} \frac{\rho_v(r')dv'}{4\pi\varepsilon_0 |\mathbf{r} - \mathbf{r}'|^2}\frac{\mathbf{r} - \mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|}$

2.2 Gauss' law $\oint_S \mathbf{D}.d\mathbf{S} = Q_{encl} = \int_{vol} \rho\, dv$

2.3 Potential Difference $V_{AB} = -\int_B^A \mathbf{E}.d\mathbf{l}$ or E – grad V

2.4 Electric Flux Density $\mathbf{D} = \varepsilon\, \mathbf{E}$

2.5 Electrostatic Energy $W_E = \frac{1}{2}\int_{\mathbf{vol}} \mathbf{D.E}\, dv = \frac{1}{2}\int_{vol} \varepsilon E^2 dv$

3.1 Biot-Savart law

Currrent Element $d\mathbf{H} = \dfrac{I\, d\mathbf{L} \times \mathbf{R}}{4\pi R^3}$

Integral Form $\mathbf{H} = \oint \dfrac{I\, d\mathbf{L} \times \mathbf{a}_R}{4\pi R^2}$

Surface Current Form $\mathbf{H} = \int_S \dfrac{\mathbf{K} \times \mathbf{a}_R\, dS}{4\pi R^2}$

Bulk Current Form $\mathbf{H} = \int_V \dfrac{\mathbf{J} \times \mathbf{a}_R\, dv}{4\pi R^2}$

3.2 Ampere's Law $\oint \mathbf{H}.d\mathbf{L} = I$

3.3 Magnetic Flux density $\mathbf{B} = \mu\, \mathbf{H}$ where, for free space, $\mu_o = 4\pi \times 10^{-7}$ H/m

3.4 Gauss' law for magnetic Field $\oint_S \mathbf{B}.d\mathbf{S} = 0$

4.1 Magnetic Potential

Scalar $V_{mag,ab} = -\int_b^a \mathbf{H}.d\mathbf{L}$

Vector $\mathbf{A} = \oint \dfrac{\mu_0 I\, d\mathbf{L}}{4\pi R}$ or $\mathbf{A} = \int_S \dfrac{\mu_0\, \mathbf{K}\, \mathrm{dS}}{4\pi R}$ or $\mathbf{A} = \int_{vol} \dfrac{\mu_0\, \mathbf{J}\, \mathrm{dv}}{4\pi R}$

5.1 Lorentz Force Law

On Charge $\mathbf{F} = Q\,(\mathbf{E} + \mathbf{v} \times \mathbf{B})$

On Current Element $d\mathbf{F} = \mathbf{J} \times \mathbf{B}\, dv$ or $dF = \mathbf{K} \times B\, dS$ or $d\mathbf{F} = I\, d\mathbf{L} \times \mathbf{B}$

Torque $d\mathbf{T} = I\, d\mathbf{S} \times \mathbf{B}$

5.2 Magnetic Energy $W_M = \frac{1}{2}\int_{\mathbf{vol}} \mathbf{B.H}\, dv = \frac{1}{2}\int_{vol} \mu H^2 dv$

5.3 Inductance $L = \dfrac{N}{I}\int_S \mathbf{B}.d\mathbf{S} = 2\dfrac{W_M}{I^2}$

6.1 Maxwell's Equations

Differential Form	Integral Form
$\nabla \times \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t}$	$\oint \mathbf{E}.dL = -\int_S \dfrac{\partial \mathbf{B}}{\partial t}.\, d\mathbf{S}$
$\nabla \times \mathbf{H} = \mathbf{J} + \dfrac{\partial \mathbf{D}}{\partial t}$	$\oint \mathbf{H}.dL = I + \int_S \dfrac{\partial \mathbf{D}}{\partial t}.\, d\mathbf{S}$
$\nabla . \mathbf{D} = \rho_v$	$\oint_S \mathbf{D}.d\mathbf{S} = \int_{vol} \rho_v\, dv$
$\nabla . \mathbf{B} = 0$	$\oint_S \mathbf{B}.d\mathbf{S} = 0$

7.1 Plane Waves

$$\frac{E_o}{H_o} = \eta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$v = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$\mathbf{S} = \mathbf{E} \times \mathbf{H}$$

For Lossy Media, Permittivity $\varepsilon = \frac{\sigma}{\omega}$ and $\varepsilon = \varepsilon' - j\varepsilon'' = \varepsilon_o(\varepsilon_r' - j\varepsilon_r'')$

Loss Tangent $\tan\theta = \frac{\sigma}{\omega\varepsilon'}$

Propagation constant

$$\alpha \approx \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon'}} \quad \textit{and} \quad \beta \approx \omega\sqrt{\mu\varepsilon'}\left[1 + \frac{1}{8}\left(\frac{\sigma}{\omega\varepsilon'}\right)^2\right]$$

7.2 Transmission Lines

Telegraphers' Eqns. $\frac{d^2V_s}{dz^2} = \gamma^2 V_s$

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$Z_0 = \sqrt{\frac{(R + j\omega L)}{(G + j\omega C)}}$$

$$Z_{in} = Z_0\left[\frac{Z_L + Z_0 \tanh\gamma l}{Z_0 + Z_L \tanh\gamma l}\right]$$

$$Z_{in} = Z_0\left[\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}\right]$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

7.3 Waveguides Coaxial $C = \frac{2\pi\varepsilon'}{\ln(r_o / r_i)}$ *and* $L = \frac{\mu}{2\pi}\ln(r_o / r_i)$

Open-Wire

$$C = \frac{\pi\varepsilon'}{\cosh^{-1}(d/2r)} \quad \textit{and} \quad L = \frac{\mu}{4}\left[\frac{1}{4} + \cosh^{-1}(d/2r)\right]$$

Microstrip $v_p = \frac{c}{\sqrt{\varepsilon_{r,eff}}}$ *where* $\varepsilon_{r,eff} = q.\varepsilon_r + (1-q).1$

Parallel Plate

$$\beta_m = \frac{\omega}{c}\sqrt{\varepsilon_r'}\sqrt{1 - \left(\frac{\omega_{cm}}{\omega}\right)^2} \quad \textit{where} \quad \omega_{cm} = \frac{m\pi}{d}\frac{c}{\sqrt{\varepsilon_r'}}$$

Rectangular For TE_{mn} mode

$$\omega_{c,mn} = \frac{c}{\sqrt{\varepsilon_r'}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

7.4 Dipole Antenna For a short current filament along z axis

$$E_r = \frac{I_o d\eta}{2\pi}\cos\theta \exp(-j\frac{2\pi r}{\lambda})\left(\frac{1}{r^2} + \frac{\lambda}{j2\pi r^3}\right)$$

$$E_\theta = \frac{I_o d\eta}{4\pi}\sin\theta \exp(-j\frac{2\pi r}{\lambda})\left(j\frac{2\pi}{\lambda r} + \frac{1}{r^2} + \frac{\lambda}{j2\pi r^3}\right)$$

$$H_\phi = \frac{I_o d}{4\pi}\sin\theta \exp(-j\frac{2\pi r}{\lambda})\left(j\frac{2\pi}{\lambda r} + \frac{1}{r^2}\right)$$

B. Electrical properties of Semiconductors

1. $p = mv = \hbar k = h/\lambda$
2. $E = h\nu = \hbar\omega$
3. $E = \frac{1}{2}mv^2 = \frac{1}{2}p^2/m = \hbar^2 k^2/2m*$
4. $m* = \hbar^2 \Big/ \frac{d^2E}{dk^2}$
5. $f_{FD} = \frac{1}{1+\exp[(E-E_F)/kT]} \approx \exp[(E_F - E)/kT] \quad if\ E >> EF$
6. $fBE =$
7. $n_o = \int_{E_c}^{\infty} f(E) N_{st}(E) dE = N_c \exp\left[\frac{-(E_c - E_F)}{kT}\right]$
8. $N_c = 2\left(\frac{2\pi m_n^* kT}{h^2}\right)^{3/2}$
9. $N_v = 2\left(\frac{2\pi m_p^* kT}{h^2}\right)^{3/2}$
10. $p_o = N_v[1 - f(E_v)] = N_v \exp[-(E_F - E_v)/kT]$
11. $n_i = N_c \exp[-(E_c - E_i)/kT]$
12. $p_i = N_v \exp[-(E_i - E_v)/kT]$
13. $n_i = \sqrt{N_c N_v}\exp[-E_g/2kT] = 2\left(\frac{2\pi kT}{h^2}\right)^{3/2}(m_m^* m_p^*)^{3/4}\exp[-E_g/2kT]$
14. $n_o = n_i \exp[(E_F - E_i)/kT]$
15. $p_o = n_i \exp[(E_i - E_v)/kT]$
16. $n_o p_o = n_i^2$

17. $n = N_c \exp[-(E_F - F_n)/kT] = n_i \exp[(F_n - E_i)/kT]$

18. $p = N_v \exp[-(F_p - E_v)/kT] = n_i \exp[(E_i - F_p)/kT]$

19. $np = n_i^2 \exp[(F_n - F_p)/kT]$

20. $\mathcal{E}(x) = -dV(x)/dx = (1/q)\, dE_i / dx$

21. $d\mathcal{E}(x)/dx = -d^2V(x)/dx^2 = \rho(x)/\varepsilon = (q/\varepsilon)[p - n + N_d^+ - N_a^-$

22. $\mu = q\bar{\tau}/m^*$

23. $v_d / dx\mathcal{E}(x) \cong \dfrac{\mu\mathcal{E}}{1 + \mu\mathcal{E}/v_{sat}}$

24. $(I_x)_{drift} = AJ_x = Aq[n\mu_n + p\mu_p)\mathcal{E}_x] = A\sigma\mathcal{E}_x$

25. $(I_x)_{drift} = AJ_x = Aq[n\mu_n + p\mu_p)\mathcal{E}_x] = A\sigma\mathcal{E}_x$

26. $J_n(x) = (J_n)_{drift} + (J_n)_{diffu} = q\mu_n n(x)\mathcal{E}_x + qD_n \dfrac{dn(x)}{dx}$

27. $J_p(x) = (J_p)_{drift} + (J_p)_{diffu} = q\mu_p p(x)\mathcal{E}_x - qD_p \dfrac{dp(x)}{dx}$

28. $J_{tot} = J_{cond} + J_{displ} = J_n + J_p + \varepsilon\, d\mathcal{E}/dt = J_n + J_p + C\, dV/dt$

29. $\dfrac{\partial p(x,t)}{\partial t} = \dfrac{\partial \delta p}{\partial t} = -\dfrac{1}{q}\dfrac{\partial J_p}{\partial x} - \dfrac{\delta p}{\tau_p}$

30. $\dfrac{\partial n(x,t)}{\partial t} = \dfrac{\partial \delta n}{\partial t} = \dfrac{1}{q}\dfrac{\partial J_n}{\partial x} - \dfrac{\delta n}{\tau_n}$

31. $L = \sqrt{D\tau}$

32. $\left(\dfrac{\partial^2 \delta n}{\partial x^2}\right)_{steady-state} = \dfrac{\delta n}{D_n \tau_n} = \dfrac{\delta n}{L_n^2}$

33. $\left(\dfrac{\partial^2 \delta p}{\partial x^2}\right)_{steady-state} = \dfrac{\delta p}{D_p \tau_p} = \dfrac{\delta p}{L_p^2}$

34. $\dfrac{D}{\mu} = \dfrac{kT}{q}$

Physical Constants		
Avogadro's Number	N_{avo}	6.022×10^{23} m^{-1}
Boltzmann's Constant	k	1.38×10^{-23} J/K = 8.617×10^{-5} eV/K
Magnitude of Electronic Charge	Q	1.602×10^{-19} Coul
Electron rest Mass	m_o	9.109×10^{-31} Kg
Permittivity of Free Space	ε	8.854×10^{-12} F/m
Planck's Constant	h	6.626×10^{-34} J sec.
Speed of Light	c	2.998×10^{8} m/sec
Thermal Voltage at 300 K	kT/q	0.02586 Volts

Material Properties			
Property	Si	Ge	GaAs
Bandgap E_g (eV)	1.12	0.66	1.42
Relative Permittivity, ε_r	11.9	16.2	12.4
Electron Mobility, μ_n (m^2/Vsec)	1450	3900	9200
Hole Mobility, μ_p (m^2/Vsec)	505	1900	320
Effective Mass of Electron, m_n^*/m_o	0.26		0.0063
Effective Mass of Hole, m_p^*/m_o	0.69		0.57
Density, d (Kg/m^3)	2.329		5.317
Lattice constant (Å)	5.431	5.646	5.653
Effective density of states – conduction band, N_c (cm^{-3})			
Effective density of states – valence band, N_v (cm^{-3})			

C. Electro-Optics

Notations are adopted from the textbook, "Optoelectronics and Photonics, Principles and Practices," Prentice Hall, S. O. Kasap, (2001).

$E(\mathbf{r}, t) = E_o \cos(\omega t - \mathbf{k} \cdot \mathbf{r} + \varphi_o)$
$\mathbf{v} = \nu\lambda = (\varepsilon_r\, \varepsilon_o\, \mu_o)^{-0.5}$
$n = c / \mathbf{v}$
$\mathbf{v}_g = d\omega / dk$
$I = \mathbf{v}\, \varepsilon_r\, \varepsilon_o\, E_o^2 / 2$
$\tan \theta_p = n_2 / n_1$
$\boldsymbol{R} = (n_2 - n_1)^2 / (n_2 + n_1)^2$
$\boldsymbol{T} = 4\, n_2\, n_1 / (n_2 + n_1)^2$
$L = m\lambda / 2$, m = 1, 2, 3 ----
$\Delta\nu \cdot \Delta t = 1$
$\sin \theta = 1.22\, \lambda / D$
$d \sin \theta = m\, \lambda$, m = 0, ±1, ±2, ----

$(4\pi n_1 a \cos \theta_m) / \lambda - \varphi_m = m\pi$
$m < 1 + (2V - \varphi) / \pi$
$V = 2\pi a\, (n_1^2 - n_2^2)^{0.5} / \lambda$
$\Delta\tau / L \approx (n_1 - n_2) / c$
$\alpha_{dB} = 10 \log (P_{in} / P_{out}) / L$

$\eta_{external} = P_{out}(\text{optical}) / IV$
$\eta_{int} = [P_{o(int)} / h\nu] / [I / e]$

$N_2 / N_1 = \exp[-(E_2 - E_1) / k_B T]$
$\Delta\nu_{1/2} = 2\, \nu_o\, (1.386\, k_B T / Mc^2)^{0.5}$
$g_{th} = \gamma - 0.5 \ln(\boldsymbol{R}_1 \boldsymbol{R}_2) / L$
$L = m\lambda_m / 2n$, m = 1, 2, 3 ----

$\eta = [I_{ph} / \mathrm{e}] / [P_{\mathrm{o}} / h\nu]$
$R = I_{ph} / P_{\mathrm{o}}$
$\Delta\sigma = \mathrm{e}\Delta \mathrm{n} (\mu_{\mathrm{e}} + \mu_{\mathrm{h}})$
$i_{\mathrm{n}} = [2\mathrm{e}(I_d + I_{ph}) B]^{0.5}$
SNR = Signal Power / Noise Power

$I_{ph} = eG_oA (l_n + \mathrm{W} + L_e)$
$I = -I_{ph} + I_o [\exp(\mathrm{e}V / nk_B\mathrm{T}) - 1]$
$\mathrm{FF} = I_mV_m / I_{sc}V_{oc}$

$n_e(\theta)^{-2} = \cos^2(\theta) / n_o^2 + \sin^2(\theta) / n_e^2$
$\varphi = 2\pi L (\mathrm{n_e} - \mathrm{n_o}) / \lambda$
$I = I_o \sin^2 (0.5\, \pi V / V_{\lambda/2})$