$i_C = I_S e^{v_{BE}/V_T}\left(1+\frac{v_{CE}}{V_A}\right)$ $i_B = \frac{i_C}{\beta}$ $i_E = (\beta+1) i_B$	$gm = \frac{\partial i_C}{\partial v_{BE}} = \frac{I_C}{v_T}$ for a BJT	$r_e = \frac{V_T}{I_E} = \frac{\alpha V_T}{I_C}$	$r_\pi = \frac{V_T}{I_B} = \frac{\beta V_T}{I_C}$	$r_o = \frac{\lvert V_A+V_{CE}\rvert}{I_C} \cong \frac{\lvert V_A\rvert}{I_C}$
$i_D = \frac{1}{2}\mu_n \frac{W}{L}(V_{GS}-V_T)^2\left(1+\frac{v_{DS}}{V_A}\right)$ $V_{OV} = V_{GS} - V_T$	$g_m = \frac{2I_D}{V_{OV}} = \sqrt{2\mu_n C_{OX}\frac{W}{L} I_D} = \mu_n C_{OX}\frac{W}{L}V_{OV}$ $g_{mb} = \chi g_m$ for a MOSFET	$I_D = \frac{1}{2} k'_n \frac{W}{L} V_{OV}^2\left(1+\frac{v_{DS}}{V_A}\right)$ $k'_n \equiv \mu_n C_{OX}$ & $V_{OV} = V_{GS} - V_T$	$r_\pi = \infty$	$r_o = \frac{\lvert V_A+V_{DS}\rvert}{I_D} \cong \frac{\lvert V_A\rvert}{I_D} = \frac{1}{\lambda I_D}$ where $\lambda = \frac{1}{V_A}$

1) The impedance "seen" looking into the Base of a NPN/PNP can be expressed, after much algebraic pain and suffering, as shown below:

$$r_b \approx r_\pi + (\beta+1)R_{Eext}\left(\frac{1}{1+\left(R_{Cext}/r_o\right)}\right) \quad (1)$$

2) In a similar fashion, the impedance "seen" looking into the emitter of a NPN/PNP, can be expressed as shown below:

$$r_{emitter} \approx \frac{R_{Bext}}{(\beta+1)} + r_e\left(\frac{r_o + R_{Cext}}{r_o + R_{Cext}/(\beta+1)}\right) \quad (3)$$

3) Finally, the impedance "seen" looking into the collector of a NPN/PNP, can be expressed as, aft way too much pain and suffering, as shown below:

$$r_c \approx r_o\left(1 + g_m\left(\frac{R_{Eext}\parallel(r_\pi + R_{Bext})}{\left(1 + R_{Bext}/r_\pi\right)}\right)\right) \quad (5)$$

4) Common Emitter (fully and partially bypassed) – the expression for the common emitter gain is represented by the following:

$$v_o/v_s = \left(v_b/v_s\right)\left(v_\pi/v_b\right)\left(v_o/v_\pi\right) = -\frac{R_{in}}{R_{in}+R_s}\,\frac{R_{Cext}}{R_{Eext}+r_e} \quad (7).$$

5) Common Base – the expression for the common base gain is represented by the followin

$$v_o/v_s = \left(v_e/v_s\right)\left(v_\pi/v_e\right)\left(v_o/v_\pi\right) = \frac{R_{in}}{R_{in}+R_s}\, g_m R_{Cext} = \frac{R_{in}}{R_{in}+R_s}\,\frac{R_{Cext}}{r_e} \quad (9)$$

6) Common Collector or Emitter Follower – the expression for the emitter follower is represented by the following:

$$v_o/v_s = \left(v_b/v_s\right)\left(v_o/v_b\right) = \frac{R_{in}}{R_{in}+R_s}\,\frac{R_{Eext}}{R_{Eext}+r_e} \quad (10)$$

Common Emitter Amplifier with R_e	Common Base Amplifier	Common Collector Amplifier
$A_v = -g_m(R_L \parallel R_c \parallel r_0)$ if $R_e = 0$ $A_v \cong -\frac{g_m}{1+g_m R_e}(R_L \parallel R_{out})$ if $R_e \neq 0$ $R_{in} = R_{BASE} \parallel (\beta+1)(r_e + R_e)$, $R_{out} = R_e \parallel r_\pi + r_0[1 + g_m(R_e \parallel r_\pi)]$	$A_v = g_m(R_C \parallel R_L)$ if $r_0 = \infty$ $R_{in} = r_e\left[\frac{R_L + r_0}{r_0 + R_L/(\beta+1)}\right]$ $R_{out} = R_e \parallel r_\pi + r_0[1 + g_m(R_e \parallel r_\pi)]$	$A_v = \frac{r_0 \parallel R_L}{(r_0 \parallel R_L) + r_e}$ $R_{in} = R_{BASE} \parallel (\beta+1)(r_e + R_L \parallel r_0)$ $R_{out} = r_0 \parallel R_e \parallel \left[r_e + \frac{R_{sig} \parallel R_{BASE}}{\beta+1}\right]$

Common Source Amplifier with R_S	Common Gate Amplifier	Common Drain Amplifier
$A_v = -g_m(r_0 \parallel R_L \parallel R_D)$ if $R_s = 0$ $A_v = -(g_m r_0)\left(\frac{R_L}{R_L + R_{out}}\right) = \frac{-g_m(R_L \parallel R_{out})}{1+(g_m+g_{mb})R_s}$ $R_{in} = R_{GATE}$, $R_{out} = r_0 + [1 + (g_m + g_{mb})r_0]R_s$	$A_v = g_m(R_D \parallel R_L)$ for $r_0 = \infty$ $R_{in} = \frac{1}{g_m + g_{mb}} + \frac{R_D \parallel R_L}{(g_m + g_{mb})r_0}$ $R_{out} = r_0 + [1 + (g_m + g_{mb})r_0]R_s$	$A_v = -\frac{g_m R'_L}{1 + g_m R'_L}$ where $R'_L = R_L \parallel r_0 \parallel \left(\frac{1}{g_{mb}}\right)$ $R_{in} = R_{GATE}$, $R_{out} = \left(\frac{1}{g_m + g_{mb}}\right) \parallel r_0 \parallel R_L$

Transfer Functions and Frequency Response

1. Let

$$A(j\varpi) = A_M \frac{\left(1+j\frac{\varpi}{z_1}\right)\left(1+j\frac{\varpi}{z_2}\right)\left(1+j\frac{\varpi}{z_3}\right)\cdots\left(1+j\frac{\varpi}{z_m}\right)}{\left(1+j\frac{\varpi}{p_1}\right)\left(1+j\frac{\varpi}{p_2}\right)\left(1+j\frac{\varpi}{p_3}\right)\cdots\left(1+j\frac{\varpi}{p_n}\right)}$$

a. <u>For the Bode Magnitude Plots</u>

$$20\log\left|A(j\varpi)\right| = 20\log\left|A_M\right| + 20\log\sqrt{1+\frac{\varpi^2}{z_1^2}} + 20\log\sqrt{1+\frac{\varpi^2}{z_2^2}} + \ldots + 20\log\sqrt{1+\frac{\varpi^2}{z_m^2}}$$
$$-\,20\log\sqrt{1+\frac{\varpi^2}{p_1^2}} - 20\log\sqrt{1+\frac{\varpi^2}{p_2^2}} - \ldots - 20\log\sqrt{1+\frac{\varpi^2}{p_n^2}}$$

b. <u>For the Bode Phase Plots</u>

$$\Phi\left[A(j\varpi)\right] = \tan^{-1} A_M + \tan^{-1}\left(\frac{\varpi}{z_1}\right) + \tan^{-1}\left(\frac{\varpi}{z_2}\right) + \ldots + \tan^{-1}\left(\frac{\varpi}{z_m}\right)$$
$$-\tan^{-1}\left(\frac{\varpi}{p_1}\right) - \tan^{-1}\left(\frac{\varpi}{p_2}\right) - \ldots - \tan^{-1}\left(\frac{\varpi}{p_n}\right)$$

---

2. Determining the 3-dB frequencies of amplifiers

(A) Poles and Zeros are Known or Can Be Easily Determined

Low-Frequency Response

$$A(s) \cong A_M F_L(s)$$

$$F_L(s) = \frac{(s+\omega z_1)(s+\omega z_2)\cdots(s+\omega z_{nL})}{(s+\omega p_1)(s+\omega p_2)\cdots(s+\omega p_{nL})}$$

If $\omega p_1 >> \omega p_2, \omega p_3, \ldots \omega z_1, \omega z_2, \ldots$

then for frequencies near the midband:

$$F_L(s) \cong \frac{s}{s+\omega p_1}, \quad \text{and} \quad \omega_L \cong \omega p_1$$

Otherwise,

$$\omega_L \cong \sqrt{\omega_{p_1}^2 + \omega_{p_2}^2 + \cdots - 2\left(\omega_{z_1}^2 + \omega_{z_2}^2 + \cdots\right)}$$

High-Frequency Response

$$A(s) \cong A_M F_H(s)$$

$$F_H(s) = \frac{(1+s/\omega z_1)(1+s/\omega z_2)\cdots(1+s/\omega z_{nH})}{(1+s/\omega p_1)(1+s/\omega p_2)\cdots(1+s/\omega p_{nH})}$$

If $\omega p_1 << \omega p_2, \omega p_3, \ldots \omega z_1, \omega z_2, \ldots$

then for frequencies near the midband:

$$F_H(s) \cong \frac{1}{1+s/\omega p_1}, \quad \text{and} \quad \omega_H \cong \omega p_1$$

Otherwise,

$$\omega_H \cong \frac{1}{\sqrt{\frac{1}{\omega_{p_1}^2} + \frac{1}{\omega_{p_2}^2} + \cdots - 2\left(\frac{1}{\omega_{z_1}^2} + \frac{1}{\omega_{z_2}^2} + \cdots\right)}}$$

(B) Poles and Zeros Cannot Be Easily Determined

Low-Frequency Response

$$F_L(s) = \frac{s^{nL} + d_1 s^{nL-1} + \cdots}{s^{nL} + e_1 s^{nL-1} + \cdots}$$

$$e_1 = \omega p_1 + \omega p_2 + \cdots + \omega p_{nL} = \sum_{i=1}^{nL} \frac{1}{C_i R_{is}}$$

If a dominant pole exists (say, P_1), then

$$\omega_L \cong \sum_{i=1}^{nL} \frac{1}{C_i R_{is}}$$

High-Frequency Response

$$F_H(s) = \frac{1 + a_1 s + a_2 s^2 + \cdots}{1 + b_1 s + b_2 s^2 + \cdots}$$

$$b_1 = \frac{1}{\omega p_1} + \frac{1}{\omega p_2} + \cdots + \frac{1}{\omega p_{nH}} = \sum_{i=1}^{nH} C_i R_{io}$$

If a dominant pole exists (say, P_1), then

$$\omega_H \cong 1 \Big/ \sum_{i=1}^{nH} C_i R_{io}$$