Applications of Electrical Noise

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Abstract—The omnipresent noise in electronic circuits and devices is generally considered undesirable. This paper describes some of the applications to which it has been put. Short descriptions of a wide variety of applications are given together with references for further details. The applications fall in four categories: applications in which noise is used as a broad-band random signal; measurements in which the random noise is used as a test signal; measurements in which noise is used as a probe into microscopic phenomena; and the applications where noise is a conceptual or theoretical tool. Many examples of applications in each of these categories are given. Some of the applications included are only of historical interest now, and a few are, as yet, only proposals.

I. Introduction

THE TITLE of this article might appear to be surprising, if not self-contradictory. For a long time, electrical engineers have studied electrical noise (or fluctuation phenomena) in circuits and devices much as physicians study a disease: not out of any affection for the subject but with a desire to eliminate it. In fact, noise is sometimes defined as "any undesirable signal." A typical chapter entitled "Noise" in an electrical engineering text would usually begin by giving such reasons for studying noise as these: "Noise contaminates the signal, setting a limit on the minimum signal strength required for proper communication; therefore, the reliability of communication could be improved, power requirement could be reduced, or larger distances could be reached by reducing noise;" or, "Noise limits the accuracy of measurements, and should be minimized to improve precision." Such statements, although true, do not fully reveal the uses to which a study of noise could be put. In this article, we propose to treat noise as a tool rather than a nuisance.

To the academically oriented, the existence of noise is reason enough to study it; it just *might* turn out to be useful. Most work on noise, however, has been motivated by more immediate applications, reduction of noise being only one of them, although a major one. This article is devoted to some of the other applications. Not very infrequently, the study of noise, or the use of noise as a tool in the study of something else, has led to significant advancements, an outstanding example being the work of Gunn [1]. He was measuring noise in semiconductor materials, while studying the properties of hot electrons, when he discovered what is now called Gunn effect. This triggered a great deal of research work because the effect has applications in making transferred-electron-type microwave oscillators and amplifiers. Of course, one should not rely on serendipity; there are plenty of other reasons for studying noise. This article will present some of them.

Most electrical engineers are aware that random-noise generators are standard laboratory instruments, so there must be *some* use for noise, but the variety of applications is often not appreciated. Furthermore, the words "applications of noise" are not synonymous with "applications of noise sources." A much broader point of view is taken here to include any use to which the study or measurement of noise can be put.

The purpose of this article is threefold. First, it is intended to generate interest in the study of noise for reasons other than as a performance limitation. Second, it is aimed toward motivating the readers to apply random-noise techniques in their own work by providing, in summary form, the case histories of past successful applications of noise. Finally, it is hoped that the article would induce some fresh thinking on how noise may be employed for other new and useful purposes.

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II. Types of Applications of Noise

For ease of study and systematization, an attempt is made in this section to classify the types of situations in which noise can be made to serve a useful purpose. One possible method of listing these situations is according to the area of application. Thus noise finds applications in biomedical engineering, circuit theory, communication systems, computers, electroacoustics, geosciences, instrumentation, physical electronics, reliability engineering, and other fields. A second method is to classify the applications by *how* noise is useful rather than *where*. This second approach will be followed here because it is more fundamental and illuminating.

There are several possible characteristics of electrical noise which make it useful in the applications discussed here. Thus there are applications based upon the fact that a noise signal can be broad-band, may arrive from a known or desired direction in space, can have a very small amplitude, or may be uncorrelated between nonoverlapping frequency bands. Very often, however, a single application is based on the existence of several of these characteristics, making it difficult to classify the applications according to these characteristics unambiguously. The noise applications are, therefore, somewhat arbitrarily clustered here into the following four categories, depending upon how the noise is employed.

A. As a Broad-Band Random Signal

A random signal can have some properties which are desirable at times, e.g., it can be incoherent and broad-band, it can be used to establish the presence or the direction of location of its source, it can simulate a random quantity, and it can be used to generate another random quantity. These properties lead to the application of random noise in electronic countermeasures, microwave heating by noise, simulation of random quantities, stochastic computing, and generation of random numbers.

B. As a Test Signal

There are many instances of measurements in which one needs a broad-band signal with precisely known properties like amplitude probability density, rms value, or autocorrelation function. Electrical random-noise generators are a convenient source of such signals, having statistical parameters which are either known in advance or can be easily manipulated. This explains the use of noise in such techniques (some of them very accurate) as measurement of impulse response, insertion loss, and linearity and intermodulation of communication equipment, as well as in noise-modulated distance-measuring radar.

C. As a Probe into Microscopic Phenomena

The use of cosmic radio noise to glean information about the sources of this noise is well known in radio astronomy. Similarly, the fact that the electrical noise is caused by the motion (or emission, or recombination, or ionizing collision, etc.) of individual carriers suggests the possibility that such microscopic processes may be studied through the fluctuations. In particular, noise measurements can be used for estimating the physical parameters which are related to these microscopic processes. This forms the basis for the application of noise in the determination of parameters like carrier lifetime in semiconductors, fundamental physical constants, and device constants, as well as in testing semiconductors for uniformity and for estimating the reliability of semiconductor devices.

D. As a Conceptual Tool

While noise has been the motivating cause for the development of new disciplines like information theory and statistical theory of communication, certain other fields, notably circuit theory, have also benefited from the study of noise. In addition to being a subject of interest in itself, noise is also useful as a vehicle for theoretical investigations and modeling of other physical systems. For example, the thermal noise of a resistor serves as a model for fluctuations in any linear dissipative system in thermodynamic equilibrium.

Specific examples of these applications are discussed in the remainder of this article.

III. Use of Noise as a Broad-Band Random Signal

A. Microwave Heating by Noise

The use of microwave power for heating in industrial processes and in microwave ovens is well known. It has been suggested that high-level microwave noise sources, like crossed-field noise generators, could be used in these applications of microwaves, especially when the load is of a nonresonant nature [2]. In such applications, the wide-band noise sources have the advantage over the resonant microwave sources (like magnetrons) that they are relatively insensitive to the load VSWR. It should be pointed out that the noise power output, available from crossed-field noise generators, is approximately the same as that available from crossed-field amplifiers (although at a lower efficiency at present). Therefore, the use of noise is not limited to lower power microwave heating applications.

B. Simulation of Random Phenomena

Noise generators have a number of practical applications in the simulation of random quantities in system evaluation and testing. One of the things that a noise signal can simulate is, of course, noise itself. This simple observation accounts for the use of noise sources in such applications as radar simulation. It is often necessary to carry out, under realistic conditions, the testing of new radar and sonar systems, and the training of personnel working with these systems, without actually placing the system in field use. This task is carried out by designing electronic simulators which generate signals resembling those encountered by the system in actual operating environment. Noise generators are an essential part of such simulators. For example, in one radar simulator [3] used for training, the random fading of signals is duplicated by modulating the signal by low-frequency random noise. The modulating signal itself is generated by cross correlating two narrow-band random-noise signals. In another radar simulator [4], the probability of detection of a target is measured by introducing noise into the videodetector. The random characteristics of noise typically observed in normal pulsed radar receivers are reproduced by adding the output of a noise source to a pulse train and rectifying it by a full-wave rectifier before applying it to the video detector. Such measurements of the visual detectability of signals in the presence of noise are not confined to radar simulation; they are also used in situations like seismic detection, where the noisy signals are recorded on charts [5]. Similar studies of the intelligibility of audio signals in the presence of noise are carried out in the evaluation of speech communication systems and in physioacoustics [6]. All such measurements may be likened to the measurement of noise figure, in that they determine the performance of a system in the presence of noise.

Noise signals are also used for simulating random vibrations in mechanical systems; the combination of a random-noise generator and a shake table is widely used to test the response of mechanical structures to random vibrations [7].

C. Source Detection and Location Through Noise Measurement

There are many well-known applications in which an object is detected or located through the measurement of the noise emitted by it. At infrared frequencies, for example, the emissions are used for the surveillance and tracking of targets, the monitoring of temperature and chemical composition, and other military and industrial applications. At microwave and radio frequencies, the noise is received to detect and identify the radio stars and atmospheric processes. At audiofrequencies, the acoustic noise from boiling liquids and vibrations of components in nuclear power reactors are picked up by sensors for fault-location or failure-detection. The literature devoted to these established technologies is very extensive [8], [9] and no attempt will be made to summarize it here. The following brief discussion is restricted to the basic principles of noise-source detection and location.

If the noise source of interest is known to move on a path, its presence can be detected by a receiver located near the path, even in the presence of a strong noise background. A practical example of this is in the detection of a passing ship by an omnidirectional hydrophone [10], [11]. (While the noise in these applications is acoustic and not electrical, the technique is included here because of its possible generality.) The effect of the passing noise source is to increase the received noise power. This increase can be identified by comparing the noise power measured over a short averaging time with a long-time average of the noise power. If the background noise level is stationary, the long-term averaging time can be increased for estimating the background noise to any desired degree of precision, so that the detectability approaches its maximum value determined by the receiver noise [10]. For nonstationary background noise, the detectability is lower and depends upon the correlation time of the background noise [11].

When the spatial location of a noise source is to be established, it is necessary to use more than one receiver and cross correlate the received signals. The method is based upon the fact that if two received signals $x(t)$ and $y(t)$ come from the same noise source $n(t)$, but with different transit-time delays T_x and T_y, their cross-correlation function is the same as the autocorrelation function of $n(t)$:

$$\phi_{xy}(\tau) = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} x(t-\tau)\, y(t)\, dt \tag{1a}$$

$$= \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} n(t-\tau-T_x)\, n(t-T_y)\, dt$$

$$= \phi_{nn}(\tau + T_x - T_y) \tag{1b}$$

and is, therefore, a maximum for $\tau = T_y - T_x$. Thus the differential transit time can be determined by an experimental measurement of $\phi_{xy}(\tau)$. If the velocity of propagation of signals is determined (possibly with the aid of a separate measurement using a known noise source), the measured value of $T_y - T_x$ yields the locus of possible locations of the unknown noise source.

For a noise source known to be along a straight line (e.g., a straight tube or a wire) two collinear detectors on the two sides of the source are sufficient for locating it [12]. The signals received at the two detectors are cross correlated and the differential delay τ required to make the cross-correlation function $\phi_{12}(\tau)$ maximum is found. A third noise detector may be used to locate a source anywhere in the plane by triangulation [13]. Such techniques have been used for locating noise sources in nuclear reactors (where again the noise is acoustic rather than electrical).

D. Self-Directional Microwave Communication

Directional antennas are used for point-to-point radio communication in order to reduce the required power and the possibility of interference with other users. When the locations of transmitting and receiving antennas are unknown or varying, it is necessary to first scan the field of view and locate the source of a pilot signal, and then to steer the antenna beam in the proper direction. An automatic method of producing properly directed beams has recently been proposed which eliminates the need for a pilot signal, scanning, and steering [14]. It is based upon the ambient noise radiated by a retrodirective antenna array. A retrodirective antenna is defined as an antenna which reradiates a beam in the same direction from which a beam is incident on it. A planar array of antenna elements connected with lines of suitable electrical length can serve as a retrodirective antenna and its retrodirective gain can be increased by inserting amplifiers in the interconnecting lines. Two such antennas which are within each other's field of view will each receive the noise output radiated by the other array, and will amplify and reradiate it towards the other. As the received noise is coherent across the receiving aperture, it is not masked by the locally generated noise. After a sufficient number of return trips around the loop between the two antennas, the signal builds up to a carrier with the radiated beams of the two antennas focused upon each other and it can then be modulated.

The spectrum of the carrier signal thus established depends upon several factors. If the voltage gain around the loop is less than one, the power spectral density of the signal tends towards a steady state, while if the gain is greater than one, it continues to build up until limiting occurs in the system. The width of the spectrum becomes narrower for larger gain, and the center frequency of the spectrum is simply the inverse of the two-way transit time between the antennas (including any interval delays) or an integral multiple of this fundamental frequency. Finally, the number of round trips required to build up to a given peak power decreases rapidly as the loop gain increases, but only slowly as the number of antenna elements in the array increases.

In many ways, the establishment of carrier in this system is similar to the buildup of oscillations in a multimode oscillator with delay. The noise signal acts as the initial excitation which grows due to feedback. The omnipresence of noise makes this application possible.

E. Applications in Electronic Countermeasures

A well-known application of high-power broad-band noise sources is in "active" jamming of radars [15] and communication equipment [16]. Radar jamming is called active if the jammer radiates a signal at the frequency of operation of the radar system, as distinguished from passive jamming which employs nonradiating devices like chaff. Active jammers themselves are either deception jammers, which radiate false echoes to confuse the radar, or noise jammers, which introduce sufficient noise in the radar receiver to mask the target echo. The noise power required for this purpose can be considerably smaller than that produced by the radar transmitter, because the jamming signal travels only one way from the target to the receiver.

Most modern radars can be tuned rapidly over several hundred megahertz so that a jammer, to be effective, must either distribute its available transmitter power uniformly over the band covered by the radar (called broad-band barrage jamming), or be capable of automatic tuning to match the radar signal frequency (called narrow-band or "spot" jamming). A third possibility is to use swept-spot jamming in which the jammer frequency is swept at a very high rate across the entire band of frequencies to be jammed such that the radar receiver is unable to recover between sweeps. Broad-band jamming is the most reliable of the three schemes but requires the largest amount of power to be successful.

The broad-band jamming signal can be generated either by a noise source centered at carrier frequency or by noise modulating a CW signal. As the noise power radiated by a jammer serves only to increase the background noise level in the receiver, the methods of reducing the effect of jamming are the same as the methods of reducing receiver noise figure. This implies that if a broad-band high-frequency noise source is used to generate the jamming signal, there is nothing that can be done to reduce jamming if the receiver is already a low-noise state-of-the-art receiver. However, if the wide-band noise was generated by frequency-modulating a CW signal by a low-frequency random-noise source, the addition of some signal-processing circuits to the receiver can reduce the effectiveness of jamming. The superiority of a high-frequency noise source stems from the fact that its output in nonoverlapping frequency bands is uncorrelated.

F. Computational Applications

Noise signals have applications in analog computation, digital computation (at least in principle), and in stochastic computation. The simulation of random processes on an analog computer requires a random signal, having prescribed statistical properties, to represent randomly varying variables, initial conditions, or parameters [17]. Noise generators used in such applications must satisfy very strict requirements in respect to stability of output, uniformity of power spectrum, wide bandwidth, and absence of hum or periodic signals. A number of such simulation studies have been made in fields like equipment failure, queuing problems in traffic control, and the nonlinear response of an airplane to random excitations [18], [19].

In digital computers, analog random-noise sources can be used to design a hard-wired source of random numbers [20], although pseudorandom-noise generators may be used in special-purpose machines while the software methods are universally used in general-purpose computers. Random-number tables can also be generated by analog noise sources; the well-known table of random numbers published by the Rand Corporation, Santa Monica, Calif., was prepared in this manner [21]. A random-frequency pulse source, producing approximately 10 000 pulses per second, was gated once per second by a constant-frequency pulse. The resulting pulses were passed through a 5-place binary counter and then converted into a decimal number by a binary-to-decimal converter. The published table is a transformed version obtained by discarding 12 of the 32 states, retaining the last digit of the decimal number, and adding pairs of digits modulo 10. The resulting table had to be tested for statistically significant bias and the circuits refined until a satisfactory table was produced.

A relatively newer application for random noise is in stochastic computers [22], [23], which are low-cost analog computers built with digital components. Stochastic computers use the probability (of switching a digital circuit) as an analog quantity and carry out the operations of multiplication, addition, integration, etc. by using digital integrated circuits that are randomly switched. The heart of such a computer is the generator of clocked random-pulse sequences (CRPS) which represent the analog quantities. In the unipolar system (which is one of the several possible forms of representation), an analog quantity is represented by the average value of a random-pulse train. Two such analog quantities can be multiplied together simply by applying the two pulse trains at the inputs of an AND gate, which results in another pulse train at the output having an average value equal to the product, as shown in Fig. 1. This is because the joint probability of two independent events is equal to the product of the probabilities of the two individual events. Similarly, integration of a quantity can be performed by a digital counter.

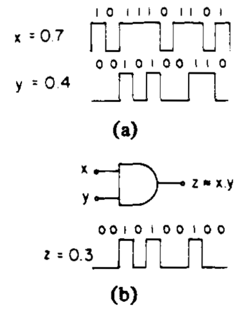

Fig. 1. (a) Unipolar representation of two quantities x and y by the average value of random pulse trains. (b) Multiplication of the analog quantities x and y by the use of an AND gate.

Stochastic computers are useful in applications such as process control where cost rather than speed or accuracy is important. The speed of these computers is limited by the width of the pulses, and the accuracy is determined by the limited number of pulses that can be used, so that the average value of the pulse train, which represents the analog variable, is subject to random-variance error.

There are several ways in which the random-pulse trains can be generated to represent an analog variable, for example, by level detection of white random noise using a trigger level proportional to the variable, and subsequent shaping by sampling of clock pulses. Alternatively, sampled flip-flops driven by noise may be used, or pseudorandom noise may be generated using shift registers.

G. Biological and Medical Applications

Noise signals are employed for biomedical purposes in at least three distinct types of applications—simulation, measurement, and therapy. Noise applications in simulation arise from the fact that fluctuations are inherent in biological systems, as they are in electronic systems. For example, the threshold of neurons at the trigger point fluctuates by 10 percent of its resting voltage. Noise signals are thus used in the simulation of the activity of neurons to produce randomness in the voltage waveforms and in the time required to generate the successive pulses [24]. The measurement of the response of biological systems to random stimuli is of interest in physiological and other studies. One system which has been extensively studied in this manner is the human auditory system [25] whose response to noise may consist of signal masking, fatigue, or damage. An interesting medical application of noise is in inducing sleep or anaesthesia [26] and in the suppression of dental pain in a technique called audioanalgesia [25]. A dental patient listens, via earphone, to relaxing music, and switches to filtered random noise on feeling pain, increasing the intensity of noise as necessary to suppress pain. While the physiological and psychological factors responsible for this phenomenon are not well understood, the collected data show that audioanalgesia has "about the same level of effectiveness as morphine" [25].

IV. Use of Noise as a Random Test Signal in Measurements

A. Measurement of Noise

Perhaps the best known of all applications of noise sources is their use in the measurement of the noisiness of electronic devices, circuits, and systems. The purpose of the measurement may be to determine the noisiness of a device, to calibrate the output of a standard noise source, to establish the limitations of a system, to optimize a design, etc., but the ultimate goal is usually the reduction of noise.[1] The literature on the subject of noise measurement is very sizable and only some surveys are referred to here.

The term "noise measurement" has been used for a number of different measurements, only three of which are briefly

[1] The critical reader could claim that this is not an "application" of noise in the true sense; the need for such characterization arises due to the problems and limitations created by noise in the first place. Pushing our analogy between noise and disease farther, this is like using virus in a vaccine to help build resistance against the virus, although in a less direct manner.

mentioned here. First, "noise measurement" refers to the measurement of the noise voltage or noise power output of a one-port device or network. Various techniques are used for carrying out the noise spectrum measurement, depending upon the frequency of measurement, the desired accuracy, the impedance level of the one-port, and other such considerations. In general, the noise output is determined by comparing it with the output of a standard noise source [27], [28]. Second, "noise measurement" is commonly used to imply the measurement of the noise figure of a linear two-port. Again there is a number of ways in which this measurement may be carried out. Most of them require that a noise source be connected at the input port and the resulting increase in the available noise power at the output port be measured [29], [30]. Third, "noise measurement" may mean the measurement of the noise modulation of the output of an oscillator or signal generator. The demodulation is then carried out in some suitable manner. The measurement of the demodulated noise power is again facilitated by comparison with standard noise-source output [31].

B. Measurement of Bandwidth

If the transfer function of a linear two-port network or system, such as a bandpass filter, is $H(\omega)$, its effective bandwidth is defined as

$$B_{\text{eff}} = \frac{1}{H_m^2} \int_0^\infty |H(\omega)|^2 \, d\omega \tag{2}$$

where H_m is the peak value of $|H(\omega)|$. The effective bandwidth is an important parameter in system design and performance calculations. Equation (2) suggests one method of determining B_{eff} by experimental measurement of the frequency response $|H(\omega)|^2$ and graphical integration of response curve. A more direct method is to use white noise (with a frequency range sufficient to encompass the passband) as the input signal so that the noise power spectrum at the output is determined by the frequency response $|H(\omega)|^2$. Measurement of the total output noise power then yields the integral of $|H(\omega)|^2$ and hence the effective bandwidth. An alternative method is to use a "standard" bandpass network. The output noise power from the unknown network is then detected and compared against that from the standard network, so that only the ratio of voltages need be measured experimentally [32].

C. Measurement of Antenna Characteristics

The experimental measurement of antenna characteristics on a test range becomes more difficult and less accurate as the size of the antenna increases. This is because the far-field criterion is more difficult to satisfy, and the possibility of spurious ground reflection is large at low elevation angles. Cosmic radio noise sources are then used for antenna measurement [33], [34] because collectively they have several different desirable properties: they always satisfy the far-field requirement, radiate unpolarized waves over a wide continuous band of frequency, have a very small angular size (essentially a point source), have a fixed position in the sky which is precisely known in many cases, have a steady power output that does not vary over short intervals and is accurately known in many cases, and cover a wide range of elevation angle and power density. The set of cosmic noise sources can, therefore, serve as an ideal test source from radio frequencies to millimeter waves (although the background radiation from the galaxy is large below 1 GHz). A black disk at a known temperature and located in the far field of the antenna has also been used like cosmic noise sources in a technique called the "artificial moon" method [34].

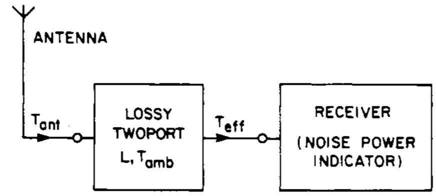

Fig. 2. Radiometric arrangement for measurement of insertion loss of a matched dissipative microwave two-port.

Several different antenna parameters can be, and have been, measured with the help of cosmic noise sources. Pointing of the electrical axis of an antenna can be carried out if the position of the radio noise source in the sky is accurately known. The true focal point of an antenna reflector can be established because the change in antenna gain is a sensitive indicator of the accuracy of focussing. The radiation pattern of an antenna can be measured accurately (because of the small angular size of noise sources) over a large dynamic range by selecting a sufficiently strong source. If, in addition, the power density of the radiation is separately determined by a small calibrated antenna and the absolute power level can be measured, absolute antenna characteristics like the aperture and the beam efficiency can also be obtained [33].

D. Measurement of Insertion Loss

The measurement of the operating noise temperature of an antenna system has been used for an accurate evaluation of the insertion loss of antenna radome material [35] and of antenna feed components between antenna aperature and receiver input port [36]. In principle, the insertion loss of any well-matched (low VSWR) two-port can be measured by the radiometric method. Fig. 2 shows a simple circuit arrangement in which a lossy two-port is present between the antenna and the noise power indicator. If 1) the connecting transmission lines are lossless, 2) the components are matched to the lines, and 3) the lossy two-port is dissipative (i.e., resistive rather than reactive), then the noise power reaching the indicator expressed as a noise temperature, is

$$T_{\text{eff}} = \frac{T_{\text{ant}}}{L} + \left(1 - \frac{1}{L}\right) T_{\text{amb}} \tag{3}$$

where T_{ant} is the antenna noise temperature, T_{amb} is the ambient (physical) temperature of the lossy two-port, and L is its loss (the ratio of input to output power). The operating noise temperature T_{eff} of the antenna is measured first with the two-port under test installed in the system, and then with the two-port replaced by one having a known insertion loss. The insertion loss of the two-port under test is then calculated from the measured noise temperatures using the relationship (3) between the insertion loss L and noise temperature T_{eff}.

E. Measurement of Microwave Signal Power

The measurement of microwave power is invariably carried out by converting the power into a suitable quantity, such as the temperature rise of a bolometer or the signal voltage across a rectifier, which can then be measured. In order to relate the measured quantity to the original signal power, it is usually necessary to calibrate the system by using a known amount of power. The power measurement is, therefore, effectively carried out by the method of substitution, in which the unknown signal is compared against a known signal. In practice, the known signal is usually at low frequency, although an RF signal is desirable for higher accuracy. A noise source can also be used to provide a reference signal, provided its output power is precisely known. A resistor is an appropriate source in this application because its thermal noise output can be calculated accurately.

The absolute power level of a CW signal received from a spacecraft has been measured by using a microwave noise signal as the reference [37]. The procedure is similar to the Y factor method of measurement of the noise figure of a linear two-port. In essence, the received output power, which consists of the system noise power and the signal power, is compared with the system noise power alone. The system noise power itself is measured by connecting at the system input a microwave thermal noise source whose output is precisely known. This technique results in a significantly increased accuracy over other methods of measurement.

F. Impulse Response Measurement

The impulse response of a linear two-port network or system can be measured with the help of stationary random noise. The random signal is applied to the system input and the resulting output is cross correlated with the input signal. It is well known [38] that if a random signal $x(t)$ with autocorrelation function $\phi_{xx}(\tau)$ is applied at the input of a linear system having an impulse response $h(t)$, the cross-correlation function between the input and the resulting output $y(t)$ is given by the convolution integral

$$\phi_{xy}(\tau) = \int_{-\infty}^{\infty} h(t)\phi_{xx}(\tau - t)\, dt. \tag{4}$$

This relationship can be used to calculate the impulse response $h(t)$ from a knowledge of ϕ_{xx} and ϕ_{xy}. For causal, lumped, linear, time-invariant systems, this calculation can be carried out algebraically [39].

In most practical situations, the choice of the input signal $x(t)$ is arbitrary. The solution of the integral equation (4) for $h(t)$ is greatly facilitated by the use of white noise as the input signal [38]. If the bandwidth of the input signal is much larger than that of the system under test, $\phi_{xx}(\tau)$ is effectively the impulse function $\delta(\tau)$ and (4) simplifies to

$$\phi_{xy}(\tau) = h(\tau). \tag{5}$$

The impulse response is thus directly measured without the need for involved calculations. Having selected white noise as the input signal, there is still a variety of random signals available to choose from. In particular, the use of a random telegraph signal (which switches randomly between two fixed amplitude levels) has the advantage that the cross correlation is easily determined, and the problems due to a limited dynamic range of the system are avoided [40].

A wide variety of linear systems have been characterized using this method, such as electrochemical electrodes [41] and nuclear reactors [42]. This method of system characterization has two major advantages: 1) the additive random noise, contributed by the system, is eliminated in the process of cross correlation, and 2) the dynamic behavior of the system is measured with a minimum of perturbation or interference with normal operation. Apart from characterization, the method is also useful as a part of the corrective loop in the design of an adaptive system. As $h(t)$ completely characterizes a linear system, a measurement of $\phi_{xy}(\tau)$ as a function of τ (or for several discrete values of τ) is sufficient for making decisions concerning the performance of a linear system. This measured system characteristic together with a predetermined criterion is usable for the continuous monitoring and readjustment of the system parameters in the design of an adaptive (self-adjusting) system [43].

G. Characterization of Nonlinear Systems

A linear system is completely described by means of its response to sinusoidal signals ("the frequency response"), or to an impulse ("the impulse response"), from which its response to any arbitrary input can be found. For nonlinear systems, the situation is much more complex. In one formulation, a complete description of the system requires a set $\{h_n\}$ of Wiener kernels [44]. The zeroth order kernel h_0 is a constant, the first-order kernel $h_1(\tau)$ is the impulse response of the best linear approximation (in the mean-square error sense) of the system, the second-order kernel $h_2(\tau_1, \tau_2)$ is the nonlinear interaction of the inputs at two time instants τ_1 and τ_2 in past, and the third-order kernel $h_3(\tau_1, \tau_2, \tau_3)$ and other higher order kernels are similarly defined. The experimental determination of Wiener kernels for a nonlinear system is a difficult process that has been attempted only infrequently, and has been carried out by using white Gaussian noise at the input to excite the system [45]. The kernels are then calculated using the expression

$$h_n(\tau_1, \tau_2, \cdots, \tau_n) = \frac{1}{n![\Phi_{xx}(f)]^n} \cdot \overline{\left\{ y(t) - \sum_{m=0}^{n-1} G_m[h_m, x(t)] \right\} \cdot x(t-\tau_1)\, x(t-\tau_2) \cdots x(t-\tau_n)} \tag{6}$$

where $x(t)$ is the input white Gaussian noise signal with power spectral density $\Phi_{xx}(f)$, resulting in the output $y(t)$, and $\{G_m\}$ is a complete set of orthogonal functionals in terms of which $y(t)$ can be expanded. This technique has been experimentally verified for electronic systems [45] and has been employed for the characterization of biological systems [46].

A very considerable amount of work in the theory of nonlinear systems has, however, been carried out using an "incomplete" description of the system, such as the response of the system to a specific class of excitations. The excitation signal waveform may be a set of sinusoids, an exponential, a staircase, etc. In particular, the response to randomly varying input signals having specified statistical characteristics is a useful description of the system [47], [48]. The effect of the nonlinear system is to transform the statistics of the input noise, and a measure of this transformation serves as a system parameter. For example, if the input and the output random signals of a nonlinear system have an almost Gaussian amplitude probability distribution, an "effective gain" of the system [49] may be defined as a function of the variance of the Gaussian distribution (or the mean-square value of the signal). This is analogous to defining, for a sinusoidal signal, a gain that is dependent upon the signal amplitude. The utility of white Gaussian noise in testing nonlinear systems stems from the fact that it contains a broad range of frequencies and a wide variety of waveforms. It, therefore, represents all possible input signals. It has been used, for example, in the testing and calibration of a nonlinear instrumentation system used for the analysis of television signals [50].

H. Measurement of Linearity and Intermodulation in a Communication Channel

The wide-band characteristic of white noise is useful for measuring the linearity and intermodulation generation in very broad-band communication equipment. Such measurements are important in evaluating the quality of multichannel communication systems. When a large number of telephone channels is to be carried by a coaxial cable or a broad-band radio link, nonlinear distortions existing in the system, if any, will introduce unwanted intermodulation products of the various components of the multiplex signal. The calculation of the intermodulation noise, so introduced, is very difficult because of the large number of channels. Since statistical properties of white noise are similar to those of a complex multichannel signal with a large number of intermittently active channels, white noise is used to simulate such a signal [51]. A method called "noise loading" is commonly used to measure the performance of communication equipment in accordance with the standard specifications. A band-limited Gaussian noise is introduced at the input into the system under test. The noise power in a test channel is measured first with all channels loaded with white noise, and then with all but the test channel loaded with white noise. The ratio of the first to the second measurement is called the noise power ratio from which the channel noise, due to intermodulation, can be calculated [52]. The spectral density of input noise can be shaped to match the signal under actual operating conditions. Instruments are commercially available to carry out this measurement [53].

Several different extensions and variations of the basic noise loading test are possible [54]. For example, the intermodulation can be computed from the spectral density of the output of a nonlinear device with a band of noise applied at the input. It is also possible to determine the order of the intermodulation noise by measuring the effect of input noise power on the intermodulation noise generated. For each decibel change in input power, the intermodulation noise changes 2 dB for a second-order system, 3 dB for a third-order system, etc.

I. Measurement of the Small-Signal Value of Nonlinear Components

In general, the measurement of the value of a circuit element requires that an external signal be applied to the element through a bridge or some other measuring system. The value of a nonlinear element, however, depends upon the magnitude of signal applied to it, so that the measured value is found for a specific signal level, and that level has to be specified. The small-signal value can usually be determined as a limit by extrapolation. A more direct method of measuring this limiting value is by employing the naturally occurring noise signal in the element itself. This method is useful even for reactive circuit elements which have only a small resistive component and hence low thermal noise. For example, Korndorf *et al.* [55] have measured the inductance of a coil with a ferromagnetic core by connecting a known capacitor across it and measuring the frequency of maximum thermal noise voltage across this resonant circuit by means of a tuned amplifier. The application is, therefore, based upon the broad-band nature of omnipresent noise.

J. Noise-Modulated Distance-Measuring Radar

One of the most interesting applications of noise is in a noise-modulated radar. In a conventional radar, the transmitted energy is periodically modulated in amplitude or frequency. The range of such radars is, therefore, limited by the repetition period of the periodic modulating signal, and there is an ambiguity in target location due to reflections from targets for which the transit time delay is greater than the repetition period. As this ambiguity is inherent in periodic modulating signals, it can be circumvented by using a nonperiodic modulating signal, for instance a noise signal. Still another attractive feature of noise-modulated radars is their noninterference with neighboring radars. With collision-avoidance radars being considered for every automobile, it is absolutely essential that the large number of radars within each other's range be protected from mutual interference. Noise modulation may be examined for this purpose.

The idea of noise-modulated radars has been around for quite some time [56], [57]. In principle, the transmitted energy may be modulated in amplitude, phase, or frequency by a random signal $x(t)$, e.g., Gaussian noise. The modulation of the signal reflected from a target is, therefore, also the same random signal $x(t + T)$, delayed by a period T, neglecting the contamination of the signal due to external noise, Doppler effect, etc. If the modulation of the outgoing carrier is cross correlated with the modulation of the returned signal, the result is the value of the autocorrelation $\phi_{xx}(\tau)$ of the random signal $x(t)$ at $\tau = T$. The range of the target can be deduced from this measurement of $\phi_{xx}(\tau = T)$, provided $\phi_{xx}(\tau)$ is a monotonic function of τ so that its inverse is a single-valued function (i.e., given $\phi_{xx}(\tau)$, one can find a unique τ). According to the Wiener–Khintchine theorem, the autocorrelation function depends upon the power spectrum of the modulating random signal. Therefore, a power spectrum can be chosen which would give a desired autocorrelation function. Merits of the various possible power spectra and modulation methods, and their limitations, have been discussed by Horton [57] who has also described the tests made on an experimental radar.

Several modified versions of the basic noise radar scheme have also been mentioned in the literature; for example, the use of naturally occurring noise from stars in order to eliminate the transmitter entirely [58], and the use of optical frequencies in a noise radar [59], and the use of noise-modulated ultrasonic radar for blood-flow measurement [60].

K. Continuous Monitoring of System Performance

The use of a noise generator for checking the system performance in manufacturing or laboratory is commonly known. The procedure can be extended to in-service monitoring of radar and communication equipment in the field. This technique has now become feasible with the development of solid-state noise sources, which have a smaller power consumption, weight, volume, radio-frequency interference, turnon time, and turnoff time, but higher noise power output and reliability than gas-discharge noise sources. Chasek [61] has described in detail the use of avalanche-diode microwave noise generators in continuous monitoring of gradually deteriorating performance parameters of radar and relay receivers, such as noise figure, distortion, gain, transmission flatness, and gain- and phase-tracking. As a result, the need for retuning or servicing the equipment is recognized before its performance becomes unacceptable. As the noise signal is very small and unrelated to all other signals, the monitoring can be carried out while the equipment is in operation, thus reducing the downtime due to checkups.

L. *Applications in Acoustic Measurements*

There are several applications of random noise in acoustic measurements. For example, the repeatability of room reverberation measurements is increased by using narrow-band random noise to generate acoustic signal rather than a pure tone because of the flat spectrum of noise in the passband. A test-signal bandwidth of approximately $20/T$ is a good compromise for optimum smoothing without losing too much spectral resolution, where T is the reverberation time [62]. Similarly, the indoor testing of the frequency response of loudspeakers using a narrow-band random noise instead of a single frequency has the advantage of smoothing out the sharp peaks and valleys introduced into the response curves by the room [63]. In another method of loudspeaker testing, broad-band random noise is considered as a voice sample for which the input and output of the loudspeaker are compared to determine the fidelity of reproduction. This method of testing is considered to be more "natural" because it resembles the manner in which a human listener would judge the fidelity [64]. Random noise is also used to obtain the diffused-field response of microphones and for testing close-talking microphones [63].

V. Use of Noise for the Measurement of Physical Parameters

A. *Measurement of Impedance and Transducer Characteristics*

The use of random noise as a test signal for measuring the characteristics of circuits and systems has already been described. In particular, it was mentioned that the small-signal value of a nonlinear inductor can be measured by measuring its resonant frequency with a known capacitor using the internally generated noise signal. In that application, the noise signal serves only as a small broad-band signal and is not used to measure something which is inherent in the noise generation mechanism. In the application under discussion now, the magnitude of internally generated noise is used as a measure of the quantity which itself is responsible for noise generation, namely the resistive part of the impedance of a two-terminal circuit or device.

According to Nyquist's theorem, any dissipative two-terminal circuit element having an impedance $Z = R + jX$ will, in thermal equilibrium at temperature T, exhibit the mean-square noise voltage and current

$$\overline{e^2} = 4kTBR \tag{7}$$

and

$$\overline{i^2} = 4kTBR/(R^2 + X^2). \tag{8}$$

These relationships can be inverted to obtain R and X from a measurement of noise voltage and current.

$$R = \frac{\overline{e^2}}{4kTB} \tag{9}$$

$$X^2 = \frac{\overline{e^2}}{\overline{i^2}} \left[1 - \frac{\overline{e^2} \cdot \overline{i^2}}{(4kTB)^2}\right]. \tag{10}$$

The sign of the reactance X can be determined by carrying out this measurement at two different frequencies while the bandwidth B is best calculated by measuring the noise of a known impedance. The measurements can be made rapidly by designing an automatic system for this purpose [65].

Obviously, this technique can be used not only for measuring impedance but also for other quantities which can be calculated from impedance. For example, the measurement of the sensitivity of a hydrophone as a function of frequency has been carried out in this way. Goncharov [66] has shown that, under certain simplifying assumptions, the current sensitivity of an electroacoustic transducer is proportional to the square root of the real part of its driving-point impedance:

$$E_I(\omega) = a \cdot (\mathrm{Re}\,[Z])^{1/2} \tag{11}$$

where a is approximately independent of frequency. Therefore, the frequency dependence of the sensitivity of a hydrophone can be determined simply by measuring the noise voltage spectrum at its terminals. As another example, this method of impedance measurement has been used for determining the temperature dependence of the dielectric constant of ferromagnetic materials [65].

The advantage of measuring impedance via noise lies in the fact that the signals involved are of very small amplitude and the small-signal value of the impedance of nonlinear devices can be measured in thermal equilibrium. In addition, the environment of the device may sometimes make it desirable to avoid an applied signal. The measurement of hydrophone impedance mentioned previously illustrates this point. When an acoustic transducer is maintained in an enclosure, the reflected waves strongly influence the value of its impedance, so that a point-by-point measurement of this impedance by a bridge circuit must be carried out with very small frequency steps (depending upon the enclosure dimensions) and is very time consuming. A more rapid measurement is possible through the spectral analysis of thermal noise.

B. *Measurement of Minority Carrier Lifetime*

Most electronic applications of semiconductors are based upon creating deviations in carrier concentrations from their equilibrium values. These "excess" concentrations build up to a steady value in the presence of a steady excitation and decay towards zero when the excitation is removed. The time constant of this exponential decay is, therefore, a parameter of fundamental importance in semiconductor work and is called the minority carrier lifetime. It has been experimentally measured in a variety of ways, some of which are based upon the measurement of noise.

The minority carrier lifetime in semiconductors can be obtained through noise measurements because the current fluctuations arise due to the generation and recombination of carriers which take place at a rate depending upon the carrier lifetime. As a result, the spectrum of current fluctuations depends upon the lifetime τ. Hill and van Vliet [67] showed that if the generation–recombination through surface states is negligible and the number of recombination centers is small compared with the equilibrium concentrations of electrons and holes in a semiconductor sample, the noise current spectrum is given by

$$S_i(\omega) = \frac{K\tau}{(1 + \omega^2\tau^2)} \ \mathrm{A}^2/\mathrm{Hz}. \tag{12}$$

This equation has been experimentally verified by measuring the noise spectra for GE samples. The carrier lifetime is, there-

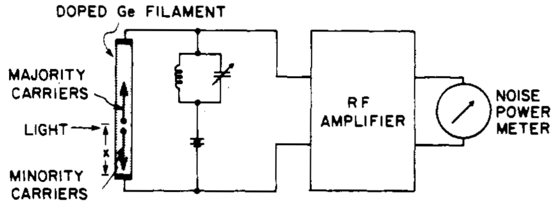

Fig. 3. Setup for measurement of minority carrier lifetime in semiconductors.

fore, directly found from the frequency at which the power spectral density drops to a half of its low-frequency value $K\tau$.

Okazaki and Oki [68] used still another method to measure the lifetime of minority carriers in germanium through the measurement of noise. A schematic diagram of their experimental arrangement is shown in Fig. 3. A source of light liberates hole–electron pairs at a spot on the surface of a germanium filament, and the RF noise in the current, collected at the ends of the filament, is measured over a narrow frequency band. The number of photogenerated minority carriers that recombine before reaching the ohmic contact depends upon the distance x between the illuminated spot and the contact collecting the minority carriers. If it is assumed that the noise power is proportional to the number of minority carriers which recombine in a mean free path, the mean-square value of noise current in a narrow bandwidth is given by

$$\overline{i^2}(x) = a \cdot n \, [1 - e^{-x/\mu E \tau}] \tag{13}$$

where a is a constant of proportionality, n is the number of injected minority carriers, μ and τ are their mobility and lifetime, and E is the electric field applied to the sample. The lifetime τ is then determined by measuring the variation of the noise power as a function of spot location x. Okazaki [69] has compared these lifetime measurements with those made using the Haynes–Shockley method to demonstrate the validity of lifetime calculations from noise measurements. Similar measurements have been carried out in silicon also [70].

C. Other Material and Carrier Transport Properties

Several other parameters have been estimated by measurements on noise. The structure and composition of ferromagnetic materials can be tested by the measurement of the spectrum of Barkhausen noise [71]. The capture cross section for the recombination of excess carriers in nickel-doped germanium has been determined by measuring the generation-recombination noise [72]. The measured amplitude distribution of the noise of a resistor has been used to calculate the time-of-flight associated with the Lorentz mean-free path [73]. The mobility, density, and lifetime of hot electrons in the minima of a many-valley semiconductor can be estimated from the noise current spectral density [74]. In all such cases, the property being measured determines (or depends upon the same parameters that determine) the characteristics of the generated noise.

D. Determination of Junction Nonuniformity

It is well known that the charge carriers moving at high speeds in a semiconductor under the influence of a large applied electric field can ionize lattice atoms by impact ionization, a phenomenon called avalanche breakdown. The ratio of the injected to the avalanche-generated current is called the multiplication factor M. The presence of small nonuniformities and inhomogeneities in a transverse plane (perpendicular to the direction of current flow) in the semiconductor influences the local value of M so that the imperfections in bulk semiconductors or junctions can be detected through the resulting variation of M over the transverse plane. This spatial variation of the local value of M is very conveniently detected through noise measurement.

The avalanche-generated current is noisy because of the randomness of the avalanche process. The mean-square value of this noise current is proportional to the cube of the multiplication factor [75], making the noise current a very sensitive measure of M. In order to detect the variation of M in the transverse plane, it is necessary to create a localized spot of avalanche ionization in this plane and scan the entire cross section of the semiconductor by this spot. Local variations of M due to nonuniformities cause large variations in the mean-square noise current which is monitored. Such an experimental arrangement, utilizing a laser beam to inject primary photocurrent at a spot, has been used to measure the spatial variation of avalanche noise and hence detect nonuniformities in silicon avalanche diodes [76].

E. Measurement of Transistor Parameters

Several different transistor parameters can be obtained by means of noise measurements on transistors. An example of such a parameter is the effective base resistance r_b which is conveniently evaluated by noise measurements (particularly if the base region is inhomogeneous) in at least four different ways:

1) Chenette and van der Ziel's method requiring the measurement of equivalent input noise resistance of the transistor [77];
2) Plumb and Chenette's method involving the minimization of the open circuit emitter flicker noise voltage [78];
3) Gibbon's method requiring transistor noise figure measurement [79]; and
4) Hsu's method of plotting the collector short-circuit noise current against the square of dc collector current [80].

The last two of these methods are briefly described here.

Gibbons proposed the evaluation of r_b by measuring the transistor noise figure as a function of source resistance at low frequencies, where flicker noise is much larger than other types of noise. For large emitter current and small collector voltage, the low-frequency noise figure becomes a minimum when the source resistance is equal to base resistance, which is thus found. The ideality factor n for the emitter junction, which appears in the junction current–voltage characteristic

$$I_E = I_s \left[\exp\left(\frac{q_e V}{nkT}\right) - 1 \right] \tag{14}$$

can also be estimated by this method. For large collector voltage and small emitter current, the low-frequency noise figure becomes a minimum for a source resistance equal to $r_b + nkT/q_e I_E$. This condition yields n if the base resistance r_b is first found, as described previously [79].

In high-gain transistors, flicker noise is not the predominant source of noise, and the base resistance can be measured by a method recently reported by Hsu [80]. For small bias currents, shot noise dominates, and the collector output short-circuit noise current, expressed as equivalent saturated diode current I_{eq}, is proportional to the base resistance and the square of collector current. Therefore, the slope of I_{eq} versus I_c^2 plot gives the base resistance. This technique can be extended to measure the transconductance g_m and the small-

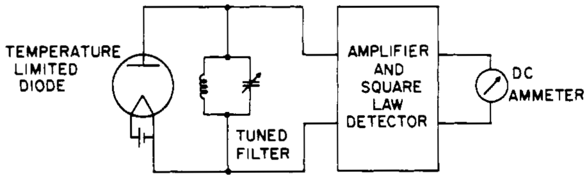

Fig. 4. Hull and Williams' apparatus for measurement of electronic charge.

signal common emitter current gain β as well. g_m can be calculated directly from base resistance and I_{eq}. For the measurement of β, an external resistance R_s is introduced in series with the base and adjusted until I_{eq} is maximum. This value of R_s, together with the base resistance and I_c, can be used to calculate β.

F. Measurement of Physical Constants

The measurement of fundamental constants like Planck's constant, the velocity of light, and the gravitational constant has occupied a central place in the physicists' interest and a sizable amount of the literature of physics. Noise measurements have been used to obtain the values of two basic constants—the electronic charge q_e and Boltzmann's constant k.

Schottky, who predicted the shot effect and derived the relationship called Schottky's theorem

$$\overline{i_n^2} = 2q_e IB \tag{15}$$

was also the first person to suggest that this relationship could be used for the determination of the electronic charge. Such a measurement was carried out by Hull and Williams in 1925, using the shot noise in a temperature-limited vacuum diode [81], [82]. Fig. 4 shows the essential features of their apparatus. They reported the average electronic charge to be 4.76×10^{-10} ESU, with the measurements having a scatter of within 2 percent of this mean value (1 coulomb = 3×10^9 ESU). Later measurements of q_e from shot noise yielded a more precise value of 4.7972×10^{-10} ESU [83].

The measurement of Boltzmann's constant k by noise was first reported by Johnson [84], who was also the first person to measure the thermal noise of a resistor. His equipment consisted of an amplifier to amplify the thermal noise voltage of a resistor connected at its input, and a thermocouple ammeter at the output to measure the mean-square noise current. Johnson calculated Boltzmann's constant by using Nyquist's theorem

$$\overline{i_n^2} = 4kTB/R \tag{16}$$

and found an average value of 1.27×10^{16} ergs/C, with a mean deviation of 13 percent. The accuracy of this measurement was later improved considerably by Ellis and Moullin [85].

G. Measurement of Temperature

As early as 1946, Lawson and Long [86], [87] proposed that the thermal noise of a resistor can be measured for an accurate estimation of very low temperatures. The most important advantage of noise thermometry lies in the fact that the thermal noise voltage generated by the resistor is independent of the composition of the resistor, previous thermal or mechanical treatments, the mass and nature of charge carriers, and the environment of the resistor other than temperature. In principle, the idea is very simple. A large resistor is kept in contact with the temperature to be measured and is connected at the input of a low-noise amplifier. The noise voltage at the output of the amplifier is a direct measure of the temperature. The method is limited by the noise contribution of the amplifier, which can be reduced by using a piezoelectric quartz crystal in place of the resistor. The advantage of quartz lies in its high Q; all of the thermal energy in a particular mode is confined to a narrow band of frequencies, and the noise voltage measurement can be made over a very small bandwidth, thus improving the signal-to-noise ratio. Temperatures down to a fraction of a kelvin can be measured in this way [88].

A noise thermometer can provide an absolute temperature standard through the use of Nyquist's theorem stated in (7), and, therefore, does not require any calibration. In practice, however, the temperature measurement is usually carried out by comparing the noise voltage of two known resistors, one at the unknown temperature and the other at a known (e.g., room) temperature [89]. Several modifications of this basic principle of noise thermometry have been employed. In one system used for the measurement of high temperatures (275 K–1275 K) in nuclear reactors, the need for a simultaneous measurement of the value of the resistor was eliminated by measuring noise power rather than noise voltage, thus improving the accuracy of measurement [90]. In another system intended for low temperatures (0.01 K–0.3 K), a superconducting quantum interference device (SQUID) was used to keep the noise contribution of the measurement system low, of the order of a millikelvin [91].

An improvement of the preceding technique has become possible with the development of superconducting weak links (Josephson junctions) which can be used for measuring temperatures of the order of (and possibly below) a millikelvin. It is based upon the fact that the measurement of a frequency can be carried out to a higher accuracy than the measurement of a voltage, so that the voltage fluctuations, due to thermal noise in a resistor, can be measured more precisely if they are first converted into frequency fluctuations. One of the important characteristics of a Josephson junction is that if a voltage V is applied across it, the resulting current alternates at a frequency

$$f = \frac{2q_e V}{h} \tag{17}$$

where h is Planck's constant, i.e., at approximately 484 GHz per mV. Random fluctuations in V will cause corresponding fluctuations in f, so that the current spectrum will have a finite line width.

In a noise thermometer constructed by Kamper and Zimmerman [92], a small resistor (of the order of 10^{-5} Ω) is biased by a 10-mA constant current source and the voltage across it is applied to a Josephson junction, causing it to oscillate at a frequency of around 40 MHz. The thermal noise voltage across the resistor frequency modulates the oscillations, generating sidebands close to the center frequency (within 1 kHz of it). In order that the thermal noise of resistor be predominant, the resistance value is chosen to be small compared with the resistances of the junction and the current supply. The spectral line width due to thermal noise broadening is given by

$$\Delta f = \frac{16\pi q_e^2 kRT}{h^2} \tag{18}$$

and is a direct measure of the temperature T. It can be calculated from the variance σ^2 of the number of cycles in a fixed gate time τ_g of a frequency counter, by using the relationship

$$\Delta f = 2\pi\tau_g\sigma^2. \tag{19}$$

The temperature is thus determined from a digital measurement of frequency.

H. Evaluation of Cathodes in Electron Tubes

The quality of the cathodes of electron tubes is conventionally evaluated in a manufacturing process through the measurement of tube parameters like the transconductance g_m. Experimental studies have shown that the shot noise of an electron tube is a more sensitive indicator of cathode life and activation than its transconductance [93]. In addition, the noise testing has no detrimental effect on the tube. Noise measurements have, therefore, been used for monitoring the aging and for optimizing the activation time of the cathode during the manufacture of electron tubes [94].

I. Prediction of Device Reliability

The reliability of electronic devices has conventionally been measured and specified in terms of statistical measures, like mean time between failures. There is experimental evidence to indicate that noise measurements may be useful as a technique for reliability testing. This technique has three distinct advantages over the conventional lifetime tests—it is nondestructive and does not use up a considerable fraction of the life of the device tested, the lifetime of a specific individual device can be measured rather than an average lifetime for a lot, and measurements do not require a long time. There are several different ways in which the measurement of the noise in a device can be employed to get information concerning the lifetime of the device.

First, the noise measurements can be used for identifying failure-prone devices because the manufacturing defects and the potential instability mechanisms become apparent through their influence on device noise. For example, it has been found that transistors with low $1/f$ noise exhibit longer life spans [95], reverse-biased p-n junction diodes having a noise power spectrum with multiple peaks undergo a more rapid degradation than those with a single-peak spectrum [96], and thin metal films having constrictions in their cross section due to scratches, notches, or pits have higher noise index than uniform films at large current densities [97].

Second, a continuous monitoring of noise can also be used to predict the impending failure of a device. For instance, it has been shown experimentally that the low-frequency $1/f$ noise output of a transistor increases by two or three orders of magnitude shortly before its failure [95]. Lifetime tests have been carried out on large batches of transistors also [98].

Taratuta [99] suggested a third and novel method for predicting lifetime and calculating failure probability of semiconductor devices, based upon the measurement of noise power spectral density of the device. The argument for relating noise spectrum to reliability goes as follows. A major cause of the failure of semiconductor devices is heating due to the random transient processes, which momentarily change the operating characteristics of the device. These transients are, therefore, accompanied by carrier density fluctuations which in turn give rise to noise. The external current through the device can be thought of as containing random pulses of different durations and magnitudes, but only the larger of these pulses are potentially destructive. By making several rather severe assumptions concerning the nature of pulses, it is possible to deduce their distribution from the measured noise spectrum for the device, and thereby to calculate the probability of the appearance of a destructive pulse in the device. Whether this method is practical is open to question; it does require making major assumptions concerning pulse shapes, rate of change of pulse duration, etc., and the calculated failure rate might turn out to be a sensitive function of these assumptions rather than the measured noise power spectrum.

J. Study of Ion Transportation Through Nerve Membranes

As in other systems, the measurement of noise generated in biological systems provides useful information about the physical mechanism involved. For example, the voltage fluctuations across the membrane of a neuron are related to the transport of ions through this permeable membrane and yield information about ion flux [100], [101]. In the resting state, the concentration of sodium, potassium, and chlorine ions inside the cell is different from that in the outside interstitial fluid, generating a potential difference across the cell membrane. An external voltage applied across the membrane causes a partial depolarization and an influx of sodium ions, which disturbance propagates along the nerve fiber. The study of membrane noise is significant not only because it influences the transmission of information within the nervous system but also because it provides a tool for studying the membrane processes on a molecular level.

The measurements of noise voltage across the membrane have shown the presence of 1) shot noise due to the motion of single ions within the membrane, 2) low-frequency $1/f$ noise related to potassium ion transport, 3) high-frequency excess noise due to fluctuations in membrane conductance, 4) burst noise, and 5) relaxation noise [100]. Each source of noise, being the result of different physical mechanisms, is a potential source of information about the various membrane properties. In addition, the membrane impedance may also be obtained from the measurement of thermal noise spectrum [101]. Such noise measurements are important because they provide a means of verifying the validity of the microscopic models of membrane processes. The attempts at interpretation and prediction of the observed noise are helping to improve the model and understanding of the ion transport through nerve membranes.

VI. Uses of Noise as a Conceptual Tool

A. Modeling and Analysis of Stochastic Systems

A number of physical systems such as communication, power, and transportation systems, having nondeterministic features, can be usefully modeled as stochastic systems. Much of the existing body of mathematical formulations and results, developed for dealing with noise in electronic circuits, devices, and systems, is then applicable to new fields. The study of random vibrations in mechanical systems is one such field that has benefited much from analogies to electrical noise and where the borrowed results have been explicitly acknowledged [102]. Another discipline is the investigation of random fluctuations in some parameters of nuclear power reactors like reactor power, temperature, and coolant flow, for the purpose of studying reactor kinetics [42] or monitoring malfunctions [103], [104].

Beyond the mere transfer of mathematical techniques, it is possible to use the concepts and principles developed with electrical noise as guides in working with other physical systems and for gaining an alternative and perhaps more intuitive viewpoint in other fields. Thermodynamics is such a field where the issues and processes are exemplified by, or lend themselves to an interpretation in terms of, electrical noise. Thus Onsager's relations for irreversible processes can be understood through noise in reciprocal two-port networks [105]. Such treatments have primarily a pedagogical value because the thermodynamic principles were already known and understood before they were interpreted in terms of electrical noise.

A rare but outstanding example of a case where the study of electrical noise preceded, motivated, and guided the development of a thermodynamic principle is the work of Callen and Welton [106] on linear dissipative systems. They observed that Nyquist's theorem (7) on thermal noise was unique in physics because it relates a property of a system in equilibrium (the voltage fluctuations) to a parameter (the electrical resistance) characterizing an irreversible process. This led them to extend the theorem and obtain a relationship between the fluctuations of the generalized forces in a linear dissipative system and the generalized resistance defined for such a system. This general formulation is capable of unifying a number of known results (on the Brownian motion, the Planck's radiation law, and the pressure fluctuations) and predicting new relationships.

B. *Noise in Circuit Theory*

One of the basic problems in the theory of electrical conduction has been to interpret impedance, which is a macroscopically observable variable, in terms of the motion of charge carriers, i.e., at microscopic level. This relationship between impedance and charge carriers can be described with the help of current fluctuations. For example, the self-inductance of an aggregate of electrons in a metallic conductor can be determined by calculating the current fluctuations [107]. More generally, it has been shown that the real part of the admittance of any linear dissipative system, as calculated from the response of the system to a generalized driving force [108], is identical to the generalized conductance obtained by Callen and Welton from fluctuation-dissipation considerations [106]. Some general theorems in circuit theory have resulted from such works. Thus Richardson [109] has shown that, for a system in equilibrium, a single impedance operator describes its transient response to an applied perturbation as well as its spontaneous transient behaviour starting from a given initial condition in the absence of external perturbation.

Thermal noise is a useful tool in theoretical investigations of circuit elements and models because 1) thermal noise is present in all dissipative systems, and 2) the available noise power depends only upon the temperature of the system. When used in conjunction with the laws of thermodynamics, noise considerations can be used to examine the validity of the circuit model of a physical system or device. Two examples of this application of noise to the verification of circuit models are described here.

The following problem in circuit theory was proposed by Penfield [110] a few years ago as a paradox. He argued that the torque generated by a universal motor (a dc motor with its field coil connected in series with its armature) is proportional to the square of the field current (which is also the armature current). Therefore, the thermal noise current of a resistor at the same temperature as the motor will generate a finite torque and deliver energy in violation of the second law of thermodynamics. This immediately suggests that the described model of the motor is oversimplified and, therefore, invalid for certain operating conditions. The resolution of the paradox indeed resulted in the development of several improved circuit models which do not lead to perpetual motion and hence have a wider range of applicability.

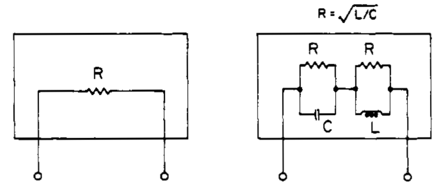

Fig. 5. Slepian–Goldner problem of black-box identification.

Similar arguments, applied to nonlinear circuit elements, suggest that there exists a relationship between noise and nonlinearity of a system because a diode cannot be used to rectify its own noise and thereby convert random noise into dc current, in violation of thermodynamic laws. Gunn [111] showed on such thermodynamic grounds that a diode with a large nonlinearity must also develop a large amount of electrical noise which imposes restrictions on the type of rectification mechanism involved. This work gave the unexpected result, which was experimentally verified by Gunn, that the equivalent circuit of a diode connected across a resistor at a lower temperature should include a constant current source, which depends upon both the idealized diode model and the load, and drives a current through the diode in the reverse (high resistance) direction.

Penfield has demonstrated the use of thermal noise concepts for proving results in network theory, and in that process has developed an analogy between frequency converters and heat engines [112]. He argues that a sinusoidal signal represents high-grade energy while the thermal noise in a resistor is low-grade energy, and the two are, therefore, analogous to work and heat, respectively, in thermodynamics. Therefore, a sinusoidally pumped three-frequency upconverter, working between two thermal noise sources (resistors) at different frequencies, is equivalent to a heat engine (or refrigerator) operating between two temperatures. Furthermore, the conversion efficiency of the lossless (reactive) upconverter, as predicted by Manley–Rowe equations, is the same as the Carnot efficiency for reversible heat engines, resulting from the second law of thermodynamics. This leads to the result that a sinusoidally pumped three-frequency upconverter obeys Manley–Rowe equations if and only if it is reversible in thermodynamic sense. This result may be generalizable to other physical systems.

Goldner [113], [114] proposed an interesting problem in circuit theory which he solved in a thought experiment using thermal noise. The problem is this: given two black boxes shown in Fig. 5, with ideal R, L, and C elements, how can the boxes be distinguished from each other by terminal measurements alone? The impedance of the boxes being identical, no transient or steady-state experiment would solve the problem. However, the two networks can be made to have different noise properties as follows. If a battery is connected across the terminals of each of the black boxes, the current in box (b) would heat only the resistor in parallel with the capacitor. Starting from uniform temperature for all elements, the two resistors in box (b) will attain different final temperatures. The noise power spectral density at the output terminals of each of the boxes can now be measured. Analysis shows that box (a) will have a larger noise power spectral density so that the two networks can be distinguished.

C. Noise and Quantum Mechanics

The study of noise in quantum electronic devices (in particular, masers and lasers) has led to the development of some analogies between quantum mechanics and the analyses of noisy circuits and systems. Heffner [115] initiated the interest in this direction when he proved the theorem that it is impossible to construct a noiseless linear (phase-preserving) amplifier. The theorem follows directly from the quantum mechanical uncertainty principle, which states that in measuring the two canonically conjugate variables E, the energy of the system, and t, the precise time at which the system possesses this energy, the uncertainties in the measured values (rms deviations from the mean in an ensemble of measurements) are related by

$$\Delta E \, \Delta t \geqq \hbar/2. \tag{20}$$

This relationship can be transformed, by substituting $E = nh\nu$ and $\phi = 2\pi\nu t$ to the form

$$\Delta n \, \Delta\phi \geqq 1/2 \tag{21}$$

where Δn is the uncertainty in the number of quanta in an oscillation, and $\Delta\phi$ is the uncertainty in its phase. If a noiseless linear amplifier precedes the instrument used to measure n and ϕ of a signal, it will increase n (the strength of the signal) without affecting the measurement of ϕ. Thus Δn can be reduced arbitrarily without influencing $\Delta\phi$, in contradiction to the uncertainty principle. This leads to the theorem that a noiseless linear amplifier cannot be constructed. Note that the connection between the uncertainty principle and the theorem is direct and does not require any models, constructs, or frameworks. In fact, the impossibility of a noiseless linear amplifier may be taken as the fundamental principle, and the uncertainty principle be treated as a manifestation of noise.

More recently, Haus [116] demonstrated that an analogy exists between the correlation functions of thermal noise and the commutator brackets of the quantum-mechanical operator amplitudes for a conservative system, because their space-time dependences are identical. This analogy allows one to obtain the commutators for a system by applying to it the fluctuation dissipation theorem (Nyquist's theorem) and solving an analogous thermal noise problem. The importance of this technique lies in unifying the classical and the quantum-mechanical methods of analyzing linear distributed systems in steady state. In the classical method, the steady-state analysis is carried out by assuming a sinusoidal excitation in the system, and the input and output signals can then be viewed as the boundary conditions imposed upon the equations describing the excitation. By contrast, the quantum-mechanical analysis is carried out using the equations of motion, which describe the evolution of the complete universe including the system, starting from an initial excitation specified by imposing the initial conditions. When the system to be analyzed quantum mechanically is an incomplete part of the universe (for example, a system of finite size, exchanging energy with the rest of the universe at its input and output ports, as in the classical method), it becomes necessary to find the commutators for the excitation amplitudes that must be imposed at the boundaries (input and output ports) in order to maintain the proper space-time dependence of commutator brackets. If no excitation was imposed at the boundaries, the initial excitation of the system would decay to a zero value with time. This situation would be similar to that of a noisy system terminated in a noiseless universe (such as loads maintained at absolute zero temperature), whereupon its energy would vanish in time, by leaking out through the ports as noise waves. The Nyquist theorem yields the noise sources that must be connected to the ports of the system to maintain it in steady state, and by analogy, also gives the commutators for excitation amplitudes at system boundaries.

VII. Epilogue

In an article of this nature, it would be difficult to list every proposed or possible application of noise exhaustively, let alone describe each in detail. The applications selected for discussion here have been taken from a variety of areas to emphasize the breadth of possibilities, including some that may never have been actually exploited (for example, Taratuta's method of predicting the reliability of semiconductor devices) and some that do not represent the current practice and state of the art (for example, the measurement of Boltzmann's constant).

The title "applications of electrical noise" has been broadly interpreted here, as is evident from the variety of ways in which noise enters into the different applications discussed. In particular, some of the "applications of noise to x" can be viewed alternatively as "applications of x to noise." For example, the measurement of electronic charge from Schottky's theorem may be thought of as the verification of the theorem (and of the assumption of independent emissions of electrons); such duality of purpose is typical of physical research [117]. Recall that when the nature of X-rays was in question, the use of the diffraction patterns from crystalline lattice was suggested for the estimation of the wavelength of X-rays. The X-rays are now commonly used to determine the structure of crystalline solids.

Discussions of the individual applications have been necessarily short, although a large number of references to the original literature is included for locating the details of specific applications. No attempt has been made here to quote every author describing a given application of noise. Where the literature on an application is extensive, reference is made to a review of the field (as, for example, in the case of membrane noise) or to a few of the earliest and the most recent papers (as in noise thermometry) to indicate the origin as well as the current status of the field. The cited references show that some of the applications of noise are as old as the discovery of electrical fluctuations and many have been known for a long time. In fact, some of the applications are only of historical interest now. At the same time, there are many newer applications attesting to a continued interest in the field.

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