Power combining efficiency and its optimisation

M.S. Gupta

Indexing terms: Power combiner, Matrix, Optimisation

Abstract: The power combining efficiency of an n-way power combiner is calculated in terms of the scattering matrix of the combiner network, and the factors affecting it are identified. The maximum value of the combining efficiency is then deduced, for a given power combiner with known network parameters, under various sets of constraints on the amplitudes and phases of the signals being combined. In each case, the signal conditions needed to achieve the maximum efficiency are determined.

1 Introduction

1.1 Motivation

Power combining techniques are commonly used at microwave and millimetre wave frequencies to deliver larger output power levels than are achievable from single active devices. A variety of power combining schemes have been described in the literature, and have been surveyed in earlier review papers [1, 2]. Many different properties of the power combiners are of interest to the circuit designer, and have been studied in detail in the literature, including the matching and isolation [3], bandwidth [4], and degradation with the failure of sources supplying the signals to be combined [5]. The present paper is concerned with another performance parameter of power combiners, namely the combining efficiency, which is of particular interest in high-power applications.

The combining efficiency of a power combiner is a measure of the extent to which the output power of the combiner approches the arithmetic sum of the powers that can be supplied by the individual sources being combined. This is an important performance specification because it governs the overall efficiency of multidevice high-power amplifiers, places a practical limit on the order of combining that can be reached by cascading lower-order combiners, and determines the highest power level that can be attained through the means of power combining. Combining efficiency is also important because it has a direct bearing on power dissipation, and therefore the need for heat removal. This paper is concerned with the determination of the maximum value of the combining efficiency of a power combiner.

The combining efficiency of a power combiner depends not only on the combiner itself (i.e. on its $[S]$ parameters), but also on the amplitudes and phases of the input signals being combined. It is well known that, for a combiner with perfect n-way symmetry (which is an idealisation), the combining efficiency is highest when the incoming signals are identical with each other in amplitude and phase. However, for an actual combiner that is not perfectly symmetric, no such maximum is discussed in the available literature, and all that has been said is that the combining efficiency will be lowered due to asymmetry. For such practical combiners, since their efficiency depends on the amplitudes and phases of the signals they receive, i.e. something external to the combiner, it is not clear how to compare two given combiners in respect of efficiency, what is the highest attainable efficiency of a given combiner, and under what conditions this highest efficiency can actually be achieved. The present paper is motivated by such questions.

Paper 8770H (E12), received 23rd July 1991

The author is with the Microwave Products Division, Industrial Electronics Group, Hughes Aircraft Company, PO Box 2940, Torrance, California 90509-2940, USA

1.2 Purpose

The maximum value of power combining efficiency has been discussed in the literature only in the context of ideal combiners having perfect symmetry. For a practical power combiner lacking in perfect symmetry, several maxima of combining efficiency exist, and are achieved under conditions of nonidentical amplitudes and phases of input signals, as will be demonstrated subsequently in this paper. The purpose of this paper is to establish the maximum value of combining efficiency in some commonly occurring situations in which the combiner is given (i.e. the scattering parameters of the combiner are invariant), while the amplitudes and/or phases of the signals being combined are adjustable (i.e. are the variables) with respect to which the efficiency is maximised. The conditions under which the combining efficiency actually reaches the maximum value are also found. Such a study of the maximum combining efficiency can be useful for several purposes:

(i) Combining efficiency maxima that depend only on the invariant combiner parameters can be employed as a figure of merit of a given combiner, or for comparing different power combiners.

(ii) Often the designer has some flexibility in adjusting the amplitudes and phases of the signals being combined; for example, the phases of the signals can be adjusted by the use of phase trimmers. In such cases, the designer may wish to determine the highest attainable combining efficiency, and the signals needed to attain it.

(iii) The actual attainment of the maximum efficiency condition may be of interest in a circuit where it is necessary to reduce the power losses occurring in the combiner network to the lowest level possible.

2 The combining efficiency

2.1 Definition of combining efficiency

Since a number of somewhat different definitions of the term 'combining efficiency' have appeared in the literature [5, 6, 7], a precise definition for it is provided briefly in this Section to avoid any ambiguity. It is then expressed in terms of the scattering parameters of the combiner network for later use.

Consider a linear $(n + 1)$ port power combiner network shown schematically in Fig. 1. Each of its n input ports is terminated in a signal source, with the

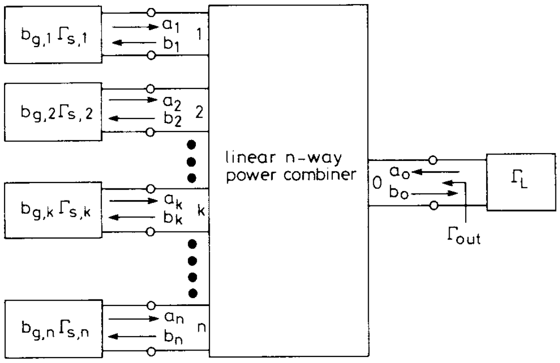

Fig. 1 *Linear $(n + 1)$ port power combiner network, terminated with linear sources and load*

source reflection coefficients $\Gamma_{s,k}$ at the kth port for $k = 1, 2, \ldots, n$, defined with respect to a real reference impedance R_0. The output port, labelled 'o', is terminated in a load having a reflection coefficient Γ_L. Such a network is completely characterised at a desired operating frequency by the $(n + 1)$ port scattering matrix

$$S \equiv \begin{bmatrix} S_{11} & S_{12} & \cdots & S_{1n} & S_{1o} \\ S_{21} & S_{22} & \cdots & S_{2n} & S_{2o} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ S_{n1} & S_{n2} & \cdots & S_{nn} & S_{no} \\ S_{o1} & S_{o2} & \cdots & S_{on} & S_{oo} \end{bmatrix} \tag{1}$$

The complex power waves a_k and b_k incident at, and reflected from, each of the ports are also defined in Fig. 1. Let $b_{g,k}$ be the complex amplitude of the power wave that would be launched on a transmission line of characteristic impedance R_0 by the source connected at the kth port.

If the n input ports of the power combiner are considered collectively as a composite input port, the combiner can be thought of as a 'two-port', and the combining efficiency is essentially the power gain of this twoport. Several different power gains can be defined [8] and will be functions of the source and load impedances in general. To define an efficiency measure independent of terminating impedances, consider the combiner under conditions of nonreflective terminations (with respect to the reference impedance R_0), so that

$$\Gamma_L = 0, \text{ and } \Gamma_{s,k} = 0, \quad k = 1, 2, \ldots, n \tag{2}$$

The value of the transducer power gain under these conditions will hereafter be called the 'combining efficiency'. Let the power available from the source at the kth port be $P_{av,k}(\Gamma_{s,k})$ and the power actually delivered to the load at the output port be $P_0(\Gamma_L)$. Then the combining efficiency is defined as

$$\eta_c \equiv \left.\frac{P_o}{\sum_{k=1}^{n} P_{av,k}}\right|_{\Gamma_L = \Gamma_{s,k} = 0;\ k = 1, 2, \ldots, n} \tag{3a}$$

$$\equiv \frac{\left|\sum_{k=1}^{n} S_{ok} b_{g,k}\right|^2}{\sum_{k=1}^{n} \left|b_{g,k}\right|^2} \tag{3b}$$

The expression for η_c shows that the efficiency depends on both the combiner parameters, and the amplitudes and phases of the signals to be combined. It has been deduced without any assumptions concerning the properties of the linear power combiner, such as losslessness, reciprocity, matching, or isolation. These properties do, however, influence the value of η_c through their influence on the values of the scattering matrix elements S_{ok} appearing in eqn. 3b. For instance, if the power combiner is known to be passive, it follows from the condition of passivity that

$$|S_{ok}|^2 \leqslant 1 - \sum_{m=1}^{n} |S_{mk}|^2 \tag{4a}$$

and

$$\sum_{k=1}^{n} |S_{ok}|^2 \leqslant 1 - |S_{oo}|^2 \tag{4b}$$

so that a lack of matching, i.e. nonzero S_{kk}, and a lack of isolation, i.e. nonzero S_{mk}, $m \neq k$, will make S_{ok} smaller, and thereby lower η_c.

2.2 Factors influencing efficiency

To separate the contribution of the various factors influencing the efficiency value, the combining efficiency in eqn. 3b can be decomposed into three factors as

$$\eta_c = \eta_1 \eta_2 \eta_3 \tag{5a}$$

where

$$\eta_1 = \frac{\left|\sum_{k=1}^{n} S_{ok} b_{g,k}\right|^2}{\sum_{k=1}^{n}\left(|b_{g,k}|^2 - \left|\sum_{m=1}^{n} S_{km} b_{g,m}\right|^2\right)} \tag{5b}$$

$$\eta_2 = \frac{1}{n}\sum_{k=1}^{n}\left(1 - \left|\sum_{m=1}^{n} S_{km}\right|^2\right) \tag{5c}$$

$$\eta_3 = \frac{n \sum_{k=1}^{n}\left(|b_{g,k}|^2 - \left|\sum_{m=1}^{n} S_{km} b_{g,m}\right|^2\right)}{\left(\sum_{k=1}^{n} |b_{g,k}|^2\right)\sum_{k=1}^{n}\left(1 - \left|\sum_{m=1}^{n} S_{km}\right|^2\right)} \tag{5d}$$

The first factor η_1 is the ratio of the actual output power to the actual input power of the combiner. It is clear from the law of conservation of energy that this term would be unity for lossless power combining*, and will be less than unity for a lossy passive combiner due to dissipation within the combiner network. The second factor η_2 is independent of the signals, and is a function only of the elements of the $n \times n$ submatrix obtained by deleting the $(n + 1)$th row and column of S in eqn. 1. It therefore depends only on the combiner matching, i.e. the diagonal elements of the submatrix, and the port isolation, i.e. the off-diagonal elements of the submatrix, and becomes unity for perfect matching and isolation. The third factor η_3 is a function of the amplitudes and phases of the signals $b_{g,k}$ to be combined. It becomes unity when all $b_{g,k}$ are identical with each other.

In conclusion, three factors contribute towards reducing the power combining efficiency of the combiner below unity:

(i) the dissipative losses in the combiner network

(ii) the lack of impedance matching at, and isolation between, each of the n ports of the combiner (with the remainder of the ports terminated in matched loads)

(iii) the variations among the amplitudes and phases of the signals to be combined.

* It is not necessary that the combiner network be lossless, but only that the combining of power be carried out losslessly.

The first two causes of power reduction are common to all microwave circuits, and the circuit designers attempt to minimise them as far as possible by methods that are well known and understood. The third cause of output power reduction is specific to power combiner circuits, and is particularly important in high power circuits which have asymmetries due to the variance of amplitude and phase among the signals to be combined. The central purpose of this paper is to carry out a detailed, quantitative analysis of efficiency optimisation in the presence of asymmetries in the power combiner and the input signals.

2.3 Expression for efficiency

To simplify the expressions encountered subsequently, the combining efficiency is expressed in terms of a transformed set of variables in this section. These variables and symbols are defined as follows:

(i) The magnitudes and angles of the generator waves and the combiner forward transmissions are defined by

$$b_{g,k} \equiv B_k \exp(j\beta_k), \quad k = 1, 2, \ldots, n \tag{6}$$

$$S_{ok} \equiv C_k \exp(j\psi_k), \; k = 1, 2, \ldots, n \tag{7}$$

(ii) The outgoing wave at the combiner output port, when this port is terminated in a nonreflective load, and its amplitude and phase, are defined by

$$b_o \equiv B_o \exp(j\beta_o) = \sum_{k=1}^{n} S_{ok} b_{g,k} \tag{8}$$

The phases of all other signals will subsequently be referred to the phase β_o of this combined output.

(iii) If the combiner were to be excited at each of its input ports by generators having unit amplitudes and unchanged phase angles β_k, the output signal would be a superposition of n signal vectors defined by

$$\begin{aligned} R_k &\equiv |R_k| \exp(j\rho_k) \equiv S_{ok}(b_{g,k}/|b_{g,k}|) \\ &= C_k \exp[j(\beta_k + \psi_k)], \quad k = 1, 2, \ldots, n \end{aligned} \tag{9}$$

(iv) With phase angles defined with respect to the previously established reference, these signal vectors can be written in Cartesian form as

$$Z_k \equiv X_k + jY_k \equiv R_k \exp(-j\beta_0), \; k = 1, 2, \ldots, n \tag{10}$$

(v) For brevity, all summations will be written without limits hereafter, and extend over $k = 1$ to n.

The combining efficiency of eqn. 3*b* can now be expressed in terms of the new variables as

$$\eta_c = \frac{|\Sigma B_k R_k|^2}{\Sigma B_k^2} = \frac{|\Sigma B_k Z_k|^2}{\Sigma B_k^2} \tag{11}$$

Since the signal phases are referred to the output signal itself

$$\begin{aligned} \Sigma B_k Y_k &= \text{Im}[\exp(-j\beta_o)\Sigma B_k R_k] \\ &= \text{Im}[\exp(-j\beta_o) b_o] = 0 \end{aligned} \tag{12}$$

Therefore eqn. 11 can be written as

$$\eta_c = \frac{(\Sigma B_k X_k)^2}{\Sigma B_k^2} \tag{13}$$

3 Maximum combining efficiency and its achievement

The expression for combining efficiency in eqn. 3*b* shows that η_c is a function of $4n$ real parameters. These include:

(i) the magnitudes $|S_{ok}|$ and the phase angles $\arg[S_{ok}]$ of the n transmission coefficients from the kth input port to the output port of the combiner, that are internal to the combiner, i.e. are determined by its design and fabrication, as well as

(ii) the amplitudes $|b_{g,k}|$ and the phase angles $\arg[b_{g,k}]$ of the n signals to be combined, that are external to the combiner, i.e. are governed by the system in which the combiner is embedded.

If some of the external parameters are variable and under the designer's control, it may be possible to adjust their values so as to maximise η_c. Moreover, the number of parameters that are variable, and the range over which they can be adjusted, will govern the maximum achievable value of η_c, and the conditions, i.e. the values of the adjustable parameters, for which the maximum efficiency is achieved. Three different maxima of η_c are deduced and discussed in the following, corresponding to three different sets of constraints, which are of interest either for conceptual or for practical purposes.

3.1 Combiner with given (fixed) parameters

The simplest case of interest is one in which the combiner is given, i.e. its S parameters are fixed, and all external parameters are variable. Then the maximum combining efficiency, attainable by varying the $2n$ adjustable parameters $b_{g,k}$, expressed as a function of the fixed parameters, is given by

$$\max[\eta_c] = \sum_{k=1}^{n} |S_{ok}|^2 \tag{14}$$

and this value of efficiency is attained if, and only if, the magnitudes and the phases of the generators are adjusted as follows

$$\text{(i)}\; |b_{g,k}| = p|S_{ok}|, \quad k = 1, 2, \ldots, n \tag{15a}$$

$$\text{(ii)}\; \arg[b_{g,k}] = \alpha - \arg[S_{ok}], \quad k = 1, 2, \ldots, n \tag{15b}$$

where p is an arbitrary scale factor and α is an arbitrary phase angle, both of them independent of k.

Proof: The proof is given here in three steps. First, we show that the expression in eqn. 14 is an upper bound of η_c; next, we show that the upper bound can actually be reached and is therefore the maximum value of η_c; finally, we determine the necessary and sufficient condition for the upper bound to be reached.

An upper bound for the efficiency follows from applying the Cauchy–Schwartz inequality [9]

$$(\Sigma B_k X_k)^2 \leqslant \Sigma B_k^2 \Sigma X_k^2 \tag{16}$$

to the right-hand side of eqn. 13. Moreover, the real part of a complex number cannot exceed the magnitude of that complex number

$$X_k \leqslant |Z_k| = |S_{ok}| \tag{17}$$

Applying eqn. 16 and eqn. 17 to eqn. 13 shows that

$$\eta_c \leqslant \Sigma X_k^2 \tag{18a}$$

$$\leqslant \Sigma |S_{ok}|^2 \tag{18b}$$

Next, we show that this upper bound is actually attainable. The equality will hold in eqn. 18*b* provided it also holds in eqns. 16 and 17. Therefore, the combining efficiency attains a value equal to its upper bound if both of the following $2n$ conditions are simultaneously satisfied:

$$\text{(i)}\; Y_k = 0 \quad \text{for } k = 1, 2, \ldots, n \tag{19a}$$

$$\text{(ii)}\; B_k = pX_k \quad \text{for } k = 1, 2, \ldots, n \tag{19b}$$

The first set of conditions, rewritten with the help of eqns. 9 and 10, requires that

$$\beta_k + \psi_k - \beta_o = 0 \quad \text{for } k = 1, 2, \ldots, n \tag{20}$$

Expressed in terms of the original variables, this is the same as the condition in eqn. 15*b*. It requires that, for each of the *n* signals being superimposed at the combiner output, the total phase angle (including its initial phase angle and the phase shift suffered in passing through the combiner) be the same. The 'arbitrary' angle α is also identified as the phase angle β_o of the output signal. If the first set of conditions has already been satisifed, the second set in eqn. 19*b* reduces to the requirement of eqn. 15*a*. Physically, this states that each of the combiner input ports should be excited with a power wave having an amplitude proportional to the magnitude of forward transmission coefficient from that port to the output port.

Two special cases of conceptual interest may be deduced from the result of this section:

(i) If the given combiner is known to have an *n* way symmetry in its parameters:

$$S_{ok} = S_{o1} \quad \text{for all } k = 2, 3, \ldots, n \tag{21}$$

the maximum attainable value of the efficiency is

$$\max[\eta_c] = n\,|S_{o1}|^2 \tag{22}$$

and is attained provided the adjustable variables satisfy the following conditions:

$$|b_{g,k}| = |b_{g,1}| \quad \text{for all } k = 2, 3, \ldots, n \tag{23a}$$

$$\arg[b_{g,k}] = \arg[b_{g,1}] \quad \text{for all } k = 2, 3, \ldots, n \tag{23b}$$

This is the well known condition for efficiency optimisation for symmetric combiners. This result can also be deduced directly by simplifying eqn. 3*b* with the help of eqn. 21, and then applying Cauchy–Schwartz inequality (eqn. 16).

(ii) If the combiner network is known to be passive, the passivity condition (eqn. 4*b*) may be applied to the upper bound in eqn. 18, leading to the upper bound

$$\eta \leqslant 1 \tag{24}$$

that is obvious from the law of conservation of energy. It is deduced here only to emphasise that the attainment of the equality in eqn. 24, which is the ultimate goal of all combiner designs, does not require either that the combiner be symmetric, or that the signals being combined be identical as in eqn. 23. For attaining unity efficiency in a lossless combiner, it is sufficient that the conditions in eqn. 15 are met and the output port is matched ($S_{00} = 0$).

The attainment of the maximum efficiency of eqn. 14 requires that both the signal generator amplitudes and the phases be adjusted in accordance with the conditions of eqn. 15. In practice, this degree of control may not be possible; for instance, either the amplitudes alone or the phases alone may be adjustable. The maximum achievable values of combining efficiency under these two conditions are deduced in the next two Sections respectively. In both cases, the maximum attainable η_c can be expected to be smaller than that in eqn. 14 because of the diminished degree of control.

3.2 Combiner with given generator wave phases

This Section considers the situation where the combiner parameters S_{ok} and the phase angles $\arg[b_{g,k}]$ of the signals being combined are fixed, whereas the signal magnitudes $|b_{g,k}|$ are adjustable. Such may be the case if the input ports of the combiner receive signals from *n* separate amplifiers with the signal parameters being governed by the gains and phase shifts of the individual amplifiers. In practice, it may be more difficult to modify and fine tune the phase shift of an amplifier after it has already been fabricated, particularly in broadband and monolithic circuits, owing to the difficulty of incorporating a variable phase shifter in a planar configuration, and owing to the thermal and mechanical requirements which will restrict the spatial displacement of individual amplifiers in a large combiner circuit. By contrast, the gain of an already fabricated amplifier may be varied easily, continuously and without deteriorating the bandwidth, in a number of ways, such as by adjusting the DC bias of active devices or by trimming some passive circuit element in the amplifier circuit. This Section considers the maximum combining efficiency that can be achieved by adjusting the gains of the individual amplifiers, and the needed distribution of signal amplitudes among the ports.

The maximum attainable value of the combining efficiency, expressed in terms of the 3*n* fixed parameters, is given by

$$\max[\eta_c] = \frac{1}{2}\sum_{k=1}^{n} |S_{ok}|^2 + \frac{1}{2}\left|\sum_{k=1}^{n} S_{ok}^2 \exp(2j \arg[b_{g,k}])\right| \tag{25}$$

and this efficiency is reached when the *n* variables (signal amplitudes) are adjusted as follows

$$|b_{g,k}| = p\,|S_{ok}| \cos(\arg[S_{ok}] + \arg[b_{g,k}] - \arg[\Sigma S_{ok} b_{g,k}]) \quad k = 1, 2, \ldots, n \tag{26}$$

where *p* is a scalar constant independent of *k*. A mathematical proof of this statement is contained in the Appendix. A physical interpretation may be ascribed to *p* by substituting eqn. 26 in eqn. 8 to obtain

$$p = \Sigma |b_{g,k}|^2 / B_o = B_o / \max[\eta_c] \tag{27}$$

The attainment of the maximum efficiency by adjusting the signal generator magnitudes has certain limitations that may be important in some cases:

(i) In high power combining, each source being combined is normally adjusted to operate at the highest possible level, and reducing its output may increase the combining efficiency, but would also decrease the overall combiner power output.

(ii) When the power output of the individual sources is adjusted, the DC power requirement of the source may not decrease in the same proportion as its output, and the efficiency of individual sources may be degraded.

(iii) Finally, the source outputs may not be adjustable at all in some cases, e.g. where the sources are injection locked, except by intentionally introducing dissipative losses in the signal path; this can lead to thermal problems and to poor overall system efficiency even if the combining efficiency is increased.

3.3 Combiner with given generator wave magnitudes

Next, consider the case where both the combiner forward transmissions S_{ok} and the generator magnitudes $|b_{g,k}|$ are fixed parameters, while the generator phase angles $\arg[b_{g,k}]$ can be varied for maximising η_c. This can be achieved in practice by introducing phase trimmers (which are essentially variable length delay lines) in the path of the signal; indeed, many types of phase trimmers are standard catalogue items. Moreover, if the phase adjustment can be carried out with very little loss, the maximisation of η_c will also maximise the efficiency of the overall system.

The maximum attainable efficiency, and the condition under which it is attained, are both expressed in terms of a scalar quantity p which is the solution of the following implicit equation

$$\sum_{k=1}^{n} |b_{g,k}| \sqrt{(p^2 |S_{ok}|^2 - |b_{g,k}|^2)} = 0 \tag{28}$$

Given all of the $2n$ parameters $|b_{g,k}|$ and $|S_{ok}|$ appearing on the left-hand side of eqn. 28, it is possible to solve for the scalar p. This equation may result in multiple solutions for p due to the following two reasons:

(i) The choice of the sign of the square root in eqn. 28 can be made arbitrarily in each of the n terms. As a result, the equation can be written as 2^{n-1} different equations, any of which could be solved to yield a value for p that satisfies eqn. 28.

(ii) The sequential numbering of the combiner input ports, and of the signal generators, can be carried out independently, leading to $n!$ ways of pairing generators with ports. Each arrangement then yields a different equation of the form of eqn. 28.

The solution for p is therefore not unique. However, corresponding to each solution for p, there exists an optimum choice of generator phase angles that yields the maximum combining efficiency. (It follows from the physical interpretation of p in eqn. 27 that, for global maximisation of combining efficiency, the smallest of all possible solutions for p should be selected.) This maximum value of η_c is given by

$$\max [\eta_c] = \frac{1}{p^2} \Sigma |b_{g,k}|^2 \tag{29}$$

and is attained when the effective generator phase angle at each of the n ports is adjusted such that

$$\arg [b_{g,k}] = \gamma - \arg [S_{ok}] + \cos^{-1}\left(\frac{|b_{g,k}|}{p|S_{ok}|}\right) \quad k = 1, 2, \ldots, n \tag{30}$$

where γ is an arbitrary constant, independent of k. Since the value of this constant is arbitrary, the phase angle at the first port can be chosen freely and only the remaining $(n-1)$ of the phase angles need be adjusted to meet this requirement. Moreover, since inverse cosine is a multiple valued function, the value at which each phase angle arg $[b_{g,k}]$ must be set is not unique. A mathematical proof of this result is contained in Appendix 6.

4 Summary and conclusions

The combining efficiency η_c of a power combiner has been defined as the 'transducer power gain' of the combiner (treating all input ports together) under conditions of nonreflective terminations at the ports. Given the scattering matrix of the combiner network, and the amplitudes and phases of the signal power waves at the input ports of the combiner, the value of η_c can be calculated from eqn. 3b. This value is influenced by dissipative losses in the combiner, matching and isolation at combiner ports, and the variations among the amplitudes and phases of signals to be combined.

Three different maximum values of η_c were found under different constraints: the maximum value of η_c is given by eqn. 14, under conditions of eqns. 15a and 15b, by eqn. 25 under conditions of eqn. 26, and by eqn. 29 under conditions of eqn. 30.

5 References

1 RUSSELL, K.J.: 'Microwave power combining techniques', *IEEE Trans.*, 1979, **MTT-27**, (5), pp. 472–478

2 CHANG, K., and SUN, C.: 'Millimeter-wave power combining techniques', *Ibid.*, 1983, **MTT-31**, (2), pp. 91–107

3 SALEH, A.A.M.: 'Planar electrically symmetric n-way hybrid power dividers/combiners', *ibid.*, 1980, **MTT-28**, (6), pp. 555–563

4 SALEH, A.A.M.: 'Computation of the frequency response of a class of symmetric n-way power dividers', *Bell Syst. Tech. J.*, 1980, **59**, (8), pp. 1493–1512

5 ERNST, R.L., CAMISA, R.L., and PRESSER, A.: 'Graceful degradation properties of matched n-port power amplifier combiner'. IEEE MTT-S International Microwave Symposium Digest, San Diego, California, June 1977, pp. 174–177

6 WAGNER, L., LATON, R.W., and WALLACE, R.N.: 'The effect of dissimiliar IMPATT diodes on power combining efficiency'. IEEE MTT-S International Microwave Symposium Digest, Boston, Mass., June 1983, pp. 489–491

7 LEE, K.J.: 'A 25 kW solid state transmitter for L-band radars'. IEEE MTT-S International Microwave Symposium Digest, Orlando, Florida, May 1979, pp. 298–302

8 VENDELIN, G.D.: 'Design of amplifiers and oscillators by the S-parameter method' (New York: Wiley-Interscience, 1982)

9 KORN, G.A., and KORN, T.M.: 'Mathematical handbook for scientists and engineers' (McGraw Hill Book Co., New York, 1968), 2nd Ed., p. 9

6 Appendix: Upper bound on efficiency

An upper bound on η_c can be deduced by employing the trigonometric identity

$$\cos^2(\rho_k - \beta_o) = \tfrac{1}{2} + \tfrac{1}{2}\cos(2\rho_k - 2\beta_o)$$

to express η_c in eqn. 18a as follows

$$\eta_c \leqslant \Sigma X_k^2 \tag{31}$$

$$= \tfrac{1}{2}\Sigma(|R_k|^2 + \text{Re}[\{R_k \exp(-j\beta_o)\}^2]) \tag{32}$$

$$= \tfrac{1}{2}\Sigma|R_k|^2 + \tfrac{1}{2}\text{Re}[\Sigma\{R_k \exp(-j\beta_o)\}^2] \tag{33}$$

$$\leqslant \tfrac{1}{2}\Sigma|R_k|^2 + \tfrac{1}{2}|\Sigma\{R_k \exp(-j\beta_o)\}^2| \tag{34}$$

$$= \tfrac{1}{2}\Sigma|R_k|^2 + \tfrac{1}{2}|\Sigma R_k^2| \tag{35}$$

where the inequality in eqn. 34 follows from the fact that the real part of a complex number is bounded by its magnitude. The expression in eqn. 35 is the desired upper bound on the combining efficiency. We next examine if, and under what conditions, η_c actually attains a value equal to the upper bound. The result in eqn. 35 will reduce to an equality provided the inequalities in eqns. 31 and 34 do. Therefore, the efficiency η_c approaches its upper bound in eqn. 35 provided the following two requirements are met

$$B_k = pX_k \quad k = 1, 2, \ldots, n \tag{36}$$

$$\text{Im}[\Sigma\{R_k \exp(-j\beta_o)\}^2] = 0 \tag{37}$$

The requirement in eqn. 37 can be written, with the help of eqn. 10, as

$$\text{Im}[\Sigma Z_k^2] = 2\Sigma X_k Y_k = 0 \tag{38}$$

If the n conditions in eqn. 36 are met, the requirement in eqn. 38 is automatically satisfied in view of eqn. 12. Therefore

(i) the requirements of eqn. 36 are sufficient for η_c to attain its upper bound, and

(ii) since the upper bound in eqn. 35 is attainable, it is the maximum value of η_c.

Reverting back to the original variables transforms the maximum value in eqn. 35 to the expression in eqn. 25, and the conditions in eqn. 36 to those in eqn. 26.

When the conditions in eqn. 36 hold, eqn. 31 reduces to the equality, so that the efficiency can also be written as

$$\eta_c = \Sigma C_k^2 \cos^2(\beta_k + \psi_k - \beta_o) \tag{39}$$

When the amplitudes B_k are the given fixed parameters and the angles β_k are the adjustable variables, the maximum value in eqn. 35 and the attainment conditions in eqn. 36 can be written in terms of B_k. For this purpose, eqn. 36 can be rewritten with the help of eqns. 9 and 10 as

$$B_k = pC_k \cos(\beta_k + \psi_k - \beta_o) \text{ where } k = 1, 2, \ldots, n \tag{40}$$

When these n conditions have been satisfied, the output wave in eqn. 8 is given by

$$B_0 = \Sigma B_k C_k \exp\{j(\beta_k + \psi_k - \beta_0)\} \tag{41}$$

Equating the real and the imaginary parts on the two sides of eqn. 41, and imposing the n conditions in eqn. 40, yields

$$\Sigma B_k^2/p = B_0 \tag{42}$$

and

$$\Sigma B_k C_k \sqrt{[1 - (B_k/pC_k)^2]} = 0 \tag{43}$$

respectively. Reverting back to the original variables transforms eqn. 43 to eqn. 28, eqn. 39 to eqn. 29, and eqn. 40 to eqn. 30.