Dynamic Range	Hardware	New Technique	Fractional Arithmetic [7]	Huang's method [8]	MRC method
64 bit range	Moduli	3,7,13,17,19,23, 29,31,37,41,47, 53,59,61	2043,2045,2047, 2051,2053,2056 2053	2043,2045, 2047,2051, 2053,2056	3,7,13,17,19,23, 29,31,37,41,47, 53,59,61
	look-up tables	554	6	26	104
	Binary Adders	0	5 (64 bits each)	43 (Total Adder bits = 674)	13 (Total Adder bits = 421)
	Comparators	0	0	6	0

In this table the number of latches required for a pipelined application are not provided since in the Conversion methods using Binary adders, the binary addition operation may have to be broken into several smaller ADD operation to match the latency of the slowest arithmetic operation. This in general may vary depending on the application.

The modulo operation using look-up tables is found to be efficient [9] for moduli values up to 6 bits. By using small moduli's it is possible to realize a dynamic range of 64 bits; using moduli which are less than or equal to 6 bits, which may be adequate for a large number of applications. The Memory compression scheme reported in [13] may further reduce the memory elements in the look-up table implementing a modulo operation but requires random logic to implement it.

V. Summary

A new technique for RNS to binary conversion *using only look-up tables* is proposed. The technique obtains the binary bits of the natural integer from the residues in a bit slice fashion by a sequence of *base extensions* to a modulus which is a power of 2. A significant reduction in latency and hardware count is possible by choosing an efficient base extension algorithm (11) in place of regular MRC. By selecting one of the "in range" moduli as a power of two further reduction in hardware and latency is feasible.

References

[1] N. S. Szabo and R. I. Tanaka, *Residue Arithmetic and its Applications to Computer Technology*. New York, McGraw-Hill, 1967.
[2] W. K. Jenkins and B. J. Leon, "The use of residue number systems in the design of finite impulse response digital filters," *IEEE Trans. Circuits Syst.*, vol. CAS-24, pp. 191–201, Apr. 1977.
[3] M. A. Soderstrand, "A high-speed low-cost recursive digital filtering using residue number arithmetic," *Proc. IEEE*, vol. 65, pp. 1065–1067, July 1977.
[4] W. K. Jenkins, "Techniques for residue-to-analog conversion for residue-Encoded Digital filters," *IEEE Trans. Circuits Syst.*, vol. CAS-25, pp. 555–562, July 1978.
[5] A. Baraniecka and G. A. Jullien, "On decoding techniques for residue number system realizations of digital signal processing hardware," *IEEE Trans. Circuits Syst.*, vol. CAS-25, pp. 935–936, Nov. 1978.
[6] A. Peled and B. Liu, "A new hardware realization of digital filters," *IEEE Trans. Acoust. Speech, Signal Processing*, vol. ASSP-22, pp. 456–465, Dec. 1974.
[7] M. A. Soderstrand, C. Vernia, and Jui-Hua Chang, "An improved residue number system digital-to-analog converter," *IEEE Trans. Circuits Syst.*, vol. CAS-30, pp. 903–906, Dec. 1983.
[8] C. H. Huang, "A fully parallel mixed-radix conversion algorithm for residue number applications," *IEEE Trans. Comput.*, vol. C-32, pp. 398–402, Apr. 1983.
[9] M. A. Bayoumi, G. A. Jullian, and W. C. Miller, "An efficient VLSI adder for DSP architectures based on RNS," in *IEEE Intl. Conf. on Acoustics, Speech, and Signal Processing*, vol. 4 of 4, pp. 1457–1460, Mar. 1985.
[10] ——, "Modes for VLSI implementation of residue number system arithmetic modulus," in *Proc. IEEE 6th Symp. on Comp. Arithmetic*, pp. 174–182, June 1983.
[11] A. P. Shenoy and R. Kumaresan, "A fast base extension method using redundant modulus in RNS," *IEEE Trans. Comput.*, to be published.
[12] F. J. Taylor, "Residue arithmetic: A Tutorial with examples," *IEEE Computer Mag.*, pp. 50–62, May 1984.
[13] C. H. Huang and F. J. Taylor, "A memory compression scheme for modular arithmetic," *IEEE Trans. Acoustics, Speech, Signal Processing*, vol. ASSP-27, pp. 608–611, Dec. 1979.
[14] M. A. Soderstrand, "A New Hardware Implementation of Modulo Adders for Residue Number Systems," *Proc. 26th Midwest Symp. on Circuits and Systems*, Mexico, pp. 412–415, Aug. 1983.
[15] A. P. Shenoy and R. Kumaresan, "A pipelined RNS to binary converter," *Proc. 29th Midwest Symp. Circuits and Systems*, Lincoln, NE, Aug. 1986.

Upper Bound on the Rate of Entropy Increase Accompanying Noise Power Flow Through Linear Systems

MADHU S. GUPTA

***Abstract*—An upper bound is established for the rate of entropy increase due to noise power flow in a non-isothermal linear network containing n independent noise sources. The bound depends on the net power flows to (or from) the noise sources collectively, the lowest noise temperature in the network, and the efficiency of a Carnot heat engine operating between the highest and the lowest noise temperature occurring in the system.**

Consider a linear lossless time-invariant n-port network, terminated at each of its ports in a dissipative and noisy linear one-port. Such a network may represent, for example, an arbitrary linear time-invariant non-isothermal dissipative network with stationary noise sources, in which each dissipative and noisy element is represented as a one port, and the remainder of the network constitutes the lossless n-port. The termination at the jth port is completely described, in steady-state and at one frequency ω, by its driving point impedance $Z_j(\omega)$ and its effective noise temperature $T_j(\omega)$, or equivalently, by the available noise power per unit bandwidth, $P_{\mathrm{av},j}(\omega) = kT_j(\omega)$, where k is Bolzmann's constant. There is some interest [1], [2] in the rate of increase of entropy in such a network due to power flow between the ports. The increase in the entropy of a heat reservoir, used for maintaining a resistive circuit element at a temperature T, can be calculated from Gibbs' equation, $\Delta E = T\Delta S$, in which ΔE is the thermal energy received by the heat reservoir, and ΔS is the resulting increase in the entropy of the reservoir [3]. The requirements of time-invariance and steady-state exclude cases where the rate of increase of entropy is time-varying, and may be related to the stability of the network [4].

Manuscript received May 20, 1987; revised February 9, 1988. This paper was recommended by Associate Editor Y. V. Genin.
The author is with the Department of Electrical Engineering and Computer Science, University of Illinois at Chicago, Chicago, IL 60680.
IEEE Log Number 8822403.

0098-4094/88/0900-1162$01.00 ©1988 IEEE

Let $P_{\text{in},j}(\omega)$ be the average noise power per unit bandwidth at frequency ω, flowing *out* of the jth port and *into* the termination $Z_j(\omega)$. From the first law of thermodynamics (energy conservation in an isolated system), it follows that

$$\sum_{j=1}^{n} P_{\text{in},j}(\omega) = 0. \tag{1}$$

From the second law of thermodynamics (nonnegative rate of entropy increase in an isolated system), it is further known that

$$\sum_{j=1}^{n} \frac{P_{\text{in},j}(\omega)}{T_j(\omega)} \geqslant 0. \tag{2}$$

Suitable ideal narrow-band filters can be postulated for arriving at these frequency-dependent forms of the thermodynamic laws. The inequality in (2) has also been deduced [2] from the properties of linear lossless networks, without explicitly invoking the second law of thermodynamics, which is not surprising since electrical networks are themselves thermodynamic systems, and the axioms of network theory (Kirchhoff's laws) are known to be consistent with the laws of thermodynamics [5]. It is, therefore, axiomatic that the rate of entropy increase is bounded from below.

The purpose of this short paper is to establish an *upper* bound to the rate of entropy increase. This bound will be expressed in terms of the following variables. Let $T_{\max}(\omega)$ and $T_{\min}(\omega)$ be the highest and the lowest temperatures among the noise temperatures $T_j(\omega)$. Then the efficiency of a Carnot (i.e., reversible) heat engine, operating between two heat reservoirs at the maximum and the minimum temperatures, is

$$\eta_C = (T_{\max} - T_{\min})/T_{\max}. \tag{3}$$

One measure of the overall level of noise power flows taking place in the network is the *net inflowing* (or outflowing) noise power, which is half the sum of the power flow magnitudes at all ports by virtue of (1):

$$P_{\text{net}}(\omega) = \sum \text{ all positive } P_{\text{in},j}(\omega) = \frac{1}{2}\sum_{j=1}^{n} |P_{\text{in},j}(\omega)|. \tag{4}$$

This quantity is a function of the lossless n-port network, as well as all $Z_j(\omega)$ and $T_j(\omega)$. The ω dependence of the various quantities will not be explicitly written in the following.

The rate of entropy increase can be written as

$$\begin{aligned}
\sum_{j=1}^{n} \frac{P_{\text{in},j}}{T_j} &= \left|\sum_{j=1}^{n} \frac{P_{\text{in},j}}{T_j}\right| && \text{from (2)} \\
&= \left|\sum_{j=1}^{n} P_{\text{in},j}\left[\frac{1}{T_j} - \frac{1}{2}\left(\frac{1}{T_{\min}} + \frac{1}{T_{\max}}\right)\right]\right| && \text{from (1)} \\
&\leqslant \sum_{j=1}^{n} |P_{\text{in},j}|\left|\frac{1}{T_j} - \frac{1}{2}\left(\frac{1}{T_{\min}} + \frac{1}{T_{\max}}\right)\right| \\
&\leqslant \sum_{j=1}^{n} |P_{\text{in},j}|\left|\frac{1}{2}\left(\frac{1}{T_{\min}} - \frac{1}{T_{\max}}\right)\right| \\
& && \text{since } T_{\min} \leqslant T_j \leqslant T_{\max} \\
&\leqslant \left(\frac{1}{T_{\min}} - \frac{1}{T_{\max}}\right) P_{\text{net}} && \text{from (4)} \\
&\leqslant \frac{P_{\text{net}}}{T_{\min}} \eta_C && \text{from (3)}.
\end{aligned} \tag{5}$$

This is the desired upper bound, and the principal result of this paper. Note that the equality in (5) holds when all terminations receiving noise power are at the minimum temperature (i.e., $T_j = T_{\min}$ for all j such that $P_{\text{in},j} > 0$), and all terminations delivering noise power are at the maximum temperature (i.e., $T_j = T_{\max}$ for all j such that $P_{\text{in},j} < 0$).

To understand the physical interpretation of the result, consider the simplest possible case of a linear lossless two port, connecting two noisy linear one ports having two different effective noise temperatures T_h and T_c at ports 1 and 2, respectively, with $T_h > T_c$. Then a flow of heat will take place, in the form of noise power P_n, from the heat reservoir at temperature T_h maintaining the noise temperature of the termination at port 1, to the heat reservoir at temperature T_c maintaining the noise temperature of the termination at port 2. The rate of decrease of the entropy of the first reservoir is P_n/T_h, while the rate of increase of the entropy of the second reservoir is P_n/T_c. The net rate of increase of entropy of the entire system is, therefore,

$$\frac{P_n}{T_c} - \frac{P_n}{T_h} = \frac{P_n}{T_c}\eta_C \tag{6}$$

where η_C is the efficiency $(T_h - T_c)/T_h$ of a Carnot engine operating between the two heat reservoirs. The bound on the rate of entropy increase, appearing in (5), takes the form

$$\sum_{j=1}^{2} \frac{P_{\text{in},j}}{T_j} \leqslant \frac{P_n + P_n}{2T_c}\eta_C \tag{7}$$

which agrees with the result in (6).

In addition to illustrating the physical interpretation of the bound found in this short paper, this example also demonstrates that, for a two port, the upper bound is actually reached and, therefore, a tighter bound cannot be found in general. Based on this example, the bound in (5) can also be interpreted as the statement that the rate of entropy increase in the given n-port system cannot exceed that in a two port in which the terminations are at the maximum and the minimum temperatures $T_{\max}$ and $T_{\min}$, and the power flow equals the net value P_{net}.

References

[1] A. S. Perelson and G. F. Oster, "On the application of network theory to non-isothermal systems," *Int. J. Circuit Theory Appl.*, vol. 4, no. 3, pp. 299–305, July 1976.

[2] J. L. Wyatt, Jr., W. M. Siebert, and H.-N. Tan, "A frequency-domain inequality for stochastic power flow in linear networks," *IEEE Trans. Circuits Syst.*, vol. CAS-31, pp. 809–814, Sept. 1984.

[3] H. B. Callen, *Thermodynamics*. New York: Wiley, 1966.

[4] R. Landauer, "Stability and entropy production in electrical circuits," *J. Stat. Phys.*, vol. 13, no. 1, pp. 1–16, July 1975.

[5] J. Meixner, Network theory and its relation to thermodynamics," presented at the *Symp. on Generalized Networks*, Polytechnic Institute of Brooklyn. Brooklyn, NY: Polytechnic, pp. 13–25, Apr. 1966.