Conductance Fluctuations in Mesoscopic Conductors at Low Temperatures

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Invited Paper

***Abstract*—This paper is a tutorial introduction to the subject of conductance fluctuations observed in mesoscopic conductors at low temperatures, and the universal conductance fluctuation (UCF) theory proposed to explain them. The discovery of the fluctuations less than a decade ago has been followed by an intensive flurry of research activity, published almost entirely in the journals of solid-state physics. This paper surveys the subject from the viewpoint of a practicing electron device engineer, with bias in favor of intuitive appeal rather than rigor, and should be helpful in understanding the primary literature on the subject. The nature of fluctuations and mesoscopic conduction are briefly introduced. Both theoretical and experimental results from the sizable literature on the subject are summarized here, emphasizing the characteristics of the fluctuations, the conditions under which they are observed, the mechanism of fluctuations, and the range of applicability of the UCF theory.**

I. Introduction

A. Motivation

As solid-state electron devices continue to shrink in size, at some stage they no longer operate in the same manner as their larger counterparts, and new features in their behavior are uncovered that require a more detailed level of description. One such threshold is reached when a classical phenomenological description of conductance, useful for larger devices of "macroscopic" dimensions, becomes inadequate. Understanding the nature of carrier transport and fluctuations in such small devices necessitates a drastically different conceptualization, and many new phenomena and principles are encountered in this study [1]–[11]. It is difficult to predict if and which of these phenomenon will be significant in the new devices that are yet to come. However, it appears certain that, as the device size shrinks, the nature of fluctuation phenomena will become increasingly diverse, and their role increasingly important. In fact, the work summarized in this paper indicates that in ultrasmall devices, the fluctuations are not merely important, they become the dominant feature.

Fluctuation of mesoscopic conductance, the subject of this paper, is a recently discovered phenomenon, unobservable in devices of larger sizes. Since it explore previously unfamiliar territory, it holds the promise of providing new insights and tools to the scientist; it also raises for the engineer the hope that its study might be useful in some new applications and devices. Some of the specific motivations for studying conductance fluctuations in mesoscopic conductors include the following.

1) The fluctuation mechanism discussed here may be relevant to some new families of nanodevices.
2) The conductance fluctuations described here may provide an explanation for, and a method for the estimation of, several low-frequency noise phenomena, including $1/f$ noise and "two-state" (or "switching") noise, observed in some types of small devices at low temperature.
3) The conductance fluctuations discussed here are sensitive to the specific locations of defects in a particular sample of conductor, and can serve as "fingerprints" of a particular sample.
4) The conductance fluctuations may provide a tool for studying transport processes and other physical phenomenon at a microscopic level, such as single-defect migration, which were previously inaccesible or difficult to observe.

B. What Are Mesoscopic Conductance Fluctuations

It seems reasonable that the study of fluctuations in ultrasmall devices should begin with the simplest possible electron device: a homogeneous sample of conductor. The simplest possible electrical characteristic of such a device is its dc small-signal conductance. Studies, both experimental and theoretical, have found that the magnitude of this conductance fluctuates under suitable conditions (low temperature, mesoscopic conductor size), and that the fluctuations are large, of the order of the value of the conductance itself. The present paper is concerned with this phenomenon.

Experimental measurements have shown that the transport coefficients (such as the conductance and magnetoresistance) of mesoscopic conductors depend on external parameters (such as applied bias voltage and magnetic field) in a seemingly random, but reproducible, manner. The value of a transport coefficient can also have a large scatter from sample to sample, or with time. The fluctuations show some unexpected characteristics, such as an increase in fluctuations upon lowering the temperature, and a universal magnitude of the rms value of fluctuations in conductance, independent of its conductance value. Further details of the observed characteristics of these fluctuations are summarized in Section VI.

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One of the earliest, and the principal, theory for understanding some of the observed fluctuation phenomena, and the only one to be discussed here, is the universal conductance fluctuation (UCF) mechanism. The UCF theory is based on treating the transport of an electron not as particle transport, but instead as the propagation of a wave, having an amplitude and a phase, that can show phase-coherent effects such as destructive or constructive intereference; the fluctuations are simply a manifestations of such quantum intereference effects in one-as well as higher-dimensional conductors. Theoretical calculations and numerical simulations of UCF have been successful in explaining many of the observed features of conductance fluctuations in ultrasmall devices under some restrictive conditions; e.g., the UCF theory is applicable only to small, weakly-disordered conductors (defined as those in which $G > q^2/h$, also called the metallic limit) at low temperatures. However, UCF theory cannot account for all experimentally observed types of fluctuations, implying that other fluctuation mechanisms may also be at work, some of them mentioned later on in Section VII. Nevertheless, UCF theory is a good starting point for the subject, because it illustrates many of the mesosopic transport issues, developes a universal description with broad applicability, and has had some degree of success in developing a theoretical understanding of the observed phenomenon.

C. History and Status of the Subject

The first experimental observation of the universal conductance fluctuations was reported in 1984 by Umbach *et al.* [12], who measured magnetoresistance of small gold rings below 1 K, and found seemingly random variations with magnetic field, rather than the periodic resistance variations that they expected. Further measurements [13] on temperature dependence verified the unexpected result that the fluctuations increased as temperature decreased. Theoretical explanation for the phenomenon was given independently by Stone [14] based on numerical simulation of diffusive transport in disordered samples, and by Al'tshuler [15] who predicted the rms value of the conductance fluctuations to be q^2/h based on transport theory. Additional theoretical studies by both Stone [16] and by Al'tshuler [17] deduced some of the expected characteristics of the fluctuations. Many of the theoretical predictions were experimentally verified by Skocpol *et al.* [18] who measured conductance fluctuations over $2\frac{1}{2}$ decades of amplitude on silicon MOSFET inversion layers of different dimensions. Feng *et al.* [19] theoretically analyzed the conduction in a two-dimensional conductor at 0 K, showing that the motion of a single impurity can give rise to conductance fluctuations with an rms value of the order of q^2/h, and proposed it as a possible mechanism for low-frequency $1/f$ noise in some low-temperature conductors.

Although the study of conductance fluctuation in ultrasmall devices is less than a decade old, the literature on the subject is already fairly large, with several hundred papers on conductance fluctuations published in the literature of physics since 1985. As is typical of the physics literature in a newly developing field, many of the theoretical studies are speculative, or are concerned with esoteric topics that are of dubious practical importance. On the other hand, in an attempt to discover new phenomena and concepts, many of the experimental studies have been carried out with exotic materials (that are rare, or extremely pure), on difficult structures (from fabrication or stability viewpoint), and under extreme operating conditions (milliKelvin temperatures, several Teslas of magnetic fields), whose engineering relevance is difficult to see, at least at the present time. The mesoscopic systems and fluctuations considered in the literature already span a wide variety. The published reports contain numerous experimental and theoretical studies of fluctuations in different parameters, as well as in various device structures, and include observations of many different types of fluctuation phenomena in ultrasmall devices, having different physical origins. Work continues on the analysis of theoretically predicted fluctuation mechanisms, identification of the various experimentally observed fluctuation phenomena, and a quantitative prediction of their composite effect in individual types of mesoscopic devices.

D. Scope and Outline of the Paper

This paper is a survey of some basic ideas extracted from the literature, presented at the level, and from the viewpoint, of a practicing electron device engineer, with the goal of conceptual understanding rather than completeness and rigor. The scope of this survey is limited in two ways as follows. Experimental studies considered here are biased in favor of familiar materials and operating conditions of possible interest to device engineers, and the fluctuations in only the conductance are discussed, ignoring those in other transport properties like magnetoresistance and Hall resistance. Theoretical studies selected for discussion are confined to those dealing with universal conductance fluctuations mechanism, and based on single-carrier models in which the carrier-to-carrier interactions are neglected; these introduce more complexity but do not alter the major conclusions.

In the context of mesoscopic conductors, the terms "fluctuations" and "conductance" have to be properly interpreted, and are therefore defined in Sections II and III respectively. Thereafter, Section IV describes the fluctuation mechanism, and Section V outlines the calculation of the rms fluctuations in conductance, with intuitive rather than rigorous emphasis. Theoretical results are presented before the experimental ones so as to provide a framework for understanding the effects of various influencing variables and the motivations of the experiments summarized in Section VI. The distinction between UCF and other low-frequency conductance fluctuation mechanisms is discussed briefly in Section VII.

II. The Nature of Fluctuations

An electron device engineer, accustomed to the engineering literature on noise in existing electron devices, may be surprised to find papers dealing with conductance "fluctuations" that report only a measurement of the entirely deterministic dependence of the magnetoconductance of a sample on magnetic field, with no signs of any fluctuating signals! The "fluctuations" that are the subject of this paper are different in several ways: they lack the features of temporal variation, irreproducibility, and self-averaging, normally connotated by the term "fluctuations." These differences are briefly explained in the present section.

A. Reproducibility and Time Variation

The traditional study of fluctuations in macroscopic electronic devices considers a variety of noise sources due to a finite temperature (thermal noise), a finite size of energy quantum (photon noise), or a finite charge per carrier (shot and partition noise). In that context, the term "fluctuation" means a random variation as a function of time, and the fluctuating quantities are the signals handled by the device, not the characterstics of the device. (When the device characteristics do fluctuate, this can usually be blamed on a fluctuating signal that modulates the otherwise invariant device characteristics.) As a result, the term "fluctuation" becomes almost synonymous with "temporal fluctuation of signal" in engineering literature. This is not the sense in which the term will be used here.

In the literature on mesoscopic devices, the term "fluctuation" has been broadly employed to include the following two phenomena:

1) a highly complex and therefore seemingly random, but entirely reproducible, dependence of some device characteristic (e.g., conductance, or magnetoresistance) on some external variable (e.g., magnetic field, or bias voltage), and
2) a macroscopically measurable change in some device characteristic (such as conductance) from one sample of the device to another, even though their macroscopic structures are identical.

Neither of these phenomena involves any temporal variations, and both are referred to as fluctuations. Indeed there is no need to distinguish between them, because they are not independent; we shall see later that these two phenomena are merely different manifestations of the same underlying mechanism.

Conductance variations due to the above mechanism can be observed also as fluctuations in time. The conductance of a mesoscopic conductor depends on the exact location of defects contained in the conductor lattice, and will therefore vary with time due to the migration of those defects within the conductor. When observed superficially, the time variation of the conductance will appear to be spontaneous and random. However, at a sufficiently detailed level of description, where the motion of individual defects is accounted for, the time-variations of conductance is of causal rather than random origin. The deterministic nature of these fluctuations becomes particularly apparent when a defect in the conductor does not migrate in an irreversible manner, but oscillates between two states, causing the conductance to vary randomly between two reproducible values. The appearance of "randomness" in time therefore depends on our choice of the depth of examination and arises from our unwillingness to consider the details of its origin.

B. Self-Averaging

It is important to point out that although mesoscopic conductance fluctuations are observed in very small conductor samples, they are not the sample-to-sample statistical fluctuations caused by manufacturing variability, which also become more apparent as a conductor becomes smaller. Since the introduction of impurities in a conductor sample is a random process (at least in the present technologies), the number of impurity atoms in multiple copies of identically prepared conductors will vary randomly, as will the sample conductance. With increasing system size (as measured by the sample volume, or the average number N of impurities in the sample), these fluctuations become relatively less important—the usual $1/\sqrt{N}$ scaling. The conductance fluctuations in mesoscopic conductors, however, have a quantum mechanical rather than a statistical origin, and one of their major distinguishing features is their unusual scaling with system size, due to the absence of self-averaging in conductance.

Self-averaging is a basic postulate of statistical mechanics that applies to an observable characteristic or parameter of a system that is extensive (i.e., one whose value is proportional to the size of the system, such as the number of impurity atoms in a homogeneous sample of conductor). The extensive variables are additive: if a given system can be treated as a composite of N smaller subsystems, the value X of an extensive parameter of the system is the sum of the values x_i of the parameter for the constituent subsystems. If x_i for the subsystems are random, then so is the value of X for the composite system, being a sum of random variables. It follows from the law of large numbers that, as N increases, X/N becomes an increasingly better estimate of the average value $\langle x_i \rangle$ over an ensemble of similar subsystems. More specifically, if the distribution of x_i is identical in each of the N subsystems, then the mean of X is

$$\langle X \rangle = N \langle x \rangle \tag{1}$$

and the fluctuations in X, denoted by $\Delta X \equiv X - \langle X \rangle$, have a variance given by

$$\langle (\Delta X)^2 \rangle = N \langle (x - \langle x \rangle)^2 \rangle. \tag{2}$$

Therefore, the rms value of fluctuations in X, normalized to the mean value of X, is given by

$$\frac{\sqrt{\langle (\Delta X)^2 \rangle}}{\langle X \rangle} \propto \frac{1}{\sqrt{N}} \frac{\sqrt{\langle (\Delta x)^2 \rangle}}{\langle x \rangle} \tag{3}$$

being inversely related to the square root of the system size. As the size of the system (or, equivalently, the ensemble average value $\langle X \rangle$ of the variable) increases, while the size of the subsystems remains constant, the fluctuations ΔX in the value of X become an increasingly smaller fraction of the ensemble avarage value $\langle X \rangle$ of the variable. This property of X is known as the self-averaging property.

The self averaging property leads to two consequences that are so "intuitively obvious" that they are apt to be taken for granted:

1) The likelihood that the variable X, measured on a particular system, deviates significantly from the average value $\langle X \rangle$ over an ensemble of similar systems can be made vanishingly small by increasing the system size.

2) The average value of a parameter is an adequate description for a particular member of the ensemble of systems if the system is statistically large.

Lest one might infer that the above does not apply to conductance because the conductance of a sample does not scale with the sample volume, we emphasize that, when viewed appropriately, the conductance of a *macroscopic* sample is an extensive, and therefore a self-averaging, parameter. Consider a three-dimensional cubic conductor of sides L. Its conductance varies as L, so that if the system size is measured in terms of the side of the cube, the conductance is effectively "extensive," and therefore self-averaging. If each side of a cubic conductor of conductance g is scaled N times, the conductance G of the resulting conductor has a variance given by

$$\frac{\sqrt{\langle(\Delta G)^2\rangle}}{\langle G\rangle} \propto \frac{1}{\sqrt{N}}\frac{\sqrt{\langle(\Delta g)^2\rangle}}{\langle g\rangle} \tag{4}$$

where the moments of G and g are each defined over their respective ensembles. More generally, for a d-dimensional conductor with side L along each dimension, it follows from Ohm's law that

$$\langle G\rangle \propto L^{d-2} \tag{5}$$

and the variance of G should scale as

$$\sqrt{\langle(\Delta G)^2\rangle}/\langle G\rangle \propto 1/L^{d/2}. \tag{6}$$

The conductance fluctuations in the *mesoscopic* samples, that are the subject of this paper, do not possess this self-averaging property. Experimental observations summarized in Section VI show that for mesoscopic conductors the rms value of fluctuations in conductance is independent of system size, and can be of the order of the average conductance itself. This is a distinguishing feature, characteristic of the mesoscopic fluctuations. Indeed, for a d-dimensional conductor to which the universal conductance fluctuation theory applies,

$$\sqrt{\langle(\Delta G)^2\rangle}/\langle G\rangle \propto L^{2-d}. \tag{7}$$

A comparison of (6) and (7) shows that, for all $d < 4$, with increasing conductor size the macroscopic conductor has a more rapid decrease in the normalized rms value of fluctuations. The reason for the different size scaling of fluctuations in mesoscopic conductors stems from their quantum mechanical origin, and the fact that the conductance depends not only on the number of imperfections, but also their locations; this is clarified in Section IV-B where the mechanism and magnitude of fluctuations are discussed.

The lack of self-averaging leads to two consequences of present interest. First, since the fluctuations can be very large, a given sample may not be adequately described by the average conductance $\langle G\rangle$ of the ensemble from which the sample is drawn—a breakdown of the average description, even when the sample size is statistically large. Second, conductance fluctuations can be observed even in large conductor samples, having as many as 10^{15} or more atoms, because they are not statistical fluctuations implied in (4). However, the sample cannot be arbitrarily large, because the relevant physical phenomenon (namely the quantum intereference effect) is observable only under certain conditions which are met when the sample is of mesoscopic size. To understand this, we next define the term "mesoscopic."

III. The Nature of Mesoscopic Conduction

The conductance fluctuation phenomenon has been observed only in very small conductors, typically with dimensions less than a tenth of micron, in the so-called mesoscopic regime. The term "mesoscopic" has often been loosely used in the literature to describe systems whose size, and properties, fall in between those of single atoms and bulk solids. This section provides a more careful definition of the term; in the process, it answers both how small must the conductor be in order to observe the conductance fluctuations, and why must it be so small. Thereafter, the conductance of a mesoscopic conductor is expressed in terms of the scattering parameters of the conductor, treated as a single scatterer.

A. Conditions for Observing Mesoscopic Phenomena

Conductance fluctuations of interest in this paper occur when the conductor satisfies the following conditions: 1) the conduction is metallic; and 2) the wavefunction of a carrier retains phase coherence in traversing through the conductor; and 3) the carrier transport is diffusive. The implications of each of these conditions, and the circumstances under which they are met, are briefly summarized here. A more detailed, tutorial exposition may be found elsewhere [20].

1) *Metallic Conduction:* The presence of defects in the conductor lattice destroys its periodicity, thus localizing the carrier wavefunctions and reducing the probability of carrier transport from one end of the conductor to the other. The large conductance, typical of metals, is therefore possible only if the length L of the conductor (measured along the direction of carrier transport) is not large compared to the coherence length ξ (which is a measure of the spatial spread of the wavefunction [21]). This condition is met provided the conductor is only "weakly disordered" so that its defect density is not too large, and gives rise to a conductance that satisfies the inequality $G \geq q^2/h$.
2) *Phase Coherence:* The wave functions of the carriers traversing the conductor loose their phase-coherence primarily due to inelastic scattering at time-varying perturbations (such as those caused by phonons) in the periodic potential of the lattice; i.e., the phase coherence length L_ϕ of carriers is essentially the mean free path $\Lambda_{\rm in}$ for inelastic scattering. To retain phase coherence requires that inelastic scattering be infrequent during carrier transit, which in turn implies that the conductor length L should be small compared to the inelastic mean free path $\Lambda_{\rm in}$. This condition can be met by reducing the number of inelastic scatterers (the phonons) through lowering the temperature. In particular, the conductor length L should be small compared to the thermal

diffusion length L_T

$$L_T = \sqrt{hD/k_B T} \tag{8}$$

over which the thermal energy spread smears out the phase coherence of carriers [22].

3) *Diffusive Transport:* The carrier motion is described as diffusive, as opposed to ballistic, when the path of the carriers in the conductor is a random walk due to scattering. For carriers to suffer numerous scatterings during their transit through the conductor, the length L of the conductor must be large compared to the mean free path Λ of the carriers. Since the inelastic scattering must remain infrequent due to condition (2) above, the scattering must be almost entirely elastic, caused by defects in the lattice. However, the defect density cannot be high due to condition (1) above. To ensure the diffusive nature of transport, therefore, the conductor length L must be restricted to the range

$$\Lambda \approx \Lambda_{\mathrm{el}} \ll L < \Lambda_{\mathrm{in}}, \quad \xi. \tag{9}$$

The elastic scattering becomes dominant ($\Lambda_{\mathrm{el}} \ll \Lambda_{\mathrm{in}}$) only at low temperatures, in the so-called "dirty conductor" regime.

B. Mesoscopic Conductance

The conductance of a macroscopic conductor can be expressed in terms of the conductivity σ

$$G = (A/L)\sigma \tag{10}$$

the conductivity in terms of carrier mobility μ

$$\sigma = nq\mu \tag{11}$$

the mobility in terms of a mean free time between collisions τ

$$\mu = q\tau/m^* \tag{12}$$

and the mean free time in terms of the mean free path Λ as

$$\tau = m^* \lambda_F \Lambda / h. \tag{13}$$

Here, A and L are the area of cross section and the length of the conductor; q, m^*, and n are the magnitude of the charge, effective mass, and density of the carriers; h is the Planck's constant, 6.63×10^{-34} Joule sec; and λ_F is the Fermi wavelength (i.e., the DeBroglie wavelength of the electrons at Fermi surface) that is related to the carrier density in a d-dimensional conductor by

$$n \approx 1/\lambda_F^d. \tag{14}$$

An intuitively appealing form of expression for G can be found by combining (10)–(14); this yields

$$G = \left(\frac{q^2}{h}\right)\left(\frac{\Lambda}{L}\right)\left(\frac{A}{\lambda_F^{(d-1)}}\right). \tag{15}$$

Note that the conductance has been expressed as a product of three ratios. The ratio (q^2/h) of the fundamental physical constants has the units of conductance, and a value equal to 0.0386 mS (or the inverse of 25812.807 Ω); it is the basic unit of conductance. The second ratio is the inverse of the length of the conductor, normalized with respect to the carrier mean free path. The third factor is the cross-sectional area of the conductor in dimensionless form, measured with Fermi wavelength as the unit of length. This form of the expression for G clarifies the relevant parameters and their natural scales.

In mesoscopic conductors, the conduction is strongly influenced by both the wave nature of the electrons, and the negligible number of inelastic scattering events within the conductor. The wave nature of electrons allows the possibility of a constructive or destructive interference between electron waves, called quantum interference effects, or the phase coherence effects. The infrequent occurrence of inelastic scattering in the conductor is essential for these effects since such scattering destroys phase coherence; it also makes the conductance highly dependent on external boundaries and terminations, a phenomenon that will be ignored here. Since inelastic scattering, which is the source of irreversibility and dissipation, does not occur within the sample, the conductor behaves very differently from a classical resistor.

One method of taking advantage of the phase coherence of the carriers in calculating the conductance of a mesoscopic conductor is Landauer's approach [23], in which conduction is treated as a scattering problem, with the transport of an electron through the conductor viewed as the propagation of an electron wavefunction through a medium. This wave is scattered by the disorders in the lattice, and the superposition of all scattered waves defines the electron transport. This situation is schematically represented in Fig. 1, where an electron suffers multiple scatterings on its way from one end of the conductor to the other. Since the wavefunction of the electron remains phase coherent as it propagates through the conductor, the entire conductor can be treated as a single scatterer, quantitatively described by its scattering parameters (namely, its reflection coefficient R and its transmission coefficient T) and the conductance is expressed in terms of these parameters. Detailed consideration shows that, for a one-dimensional conductor, when the leads are idealized and are included along with the sample, the conductance of the entire system can be written as

$$G_{\mathrm{sys}} = (q^2/h)T. \tag{16}$$

This one of the several versions of Landauer's formula [24].

In a multi-dimensional conductor, the electron waves can arrive at one end of the sample, and leave at the other end, in various quantized directions, and follow different paths within the sample, suffering many elastic collisions on the way; each such possible path of a carrier through the conductor is called a "channel," and is schematically shown in Fig. 1. Since a channel connects two modes, one at each end of the conductor, the total number of channels N_{ch} can be estimated as follows.

$$N_{\mathrm{ch}} \sim N_{\mathrm{mode}}^2 \sim (Wk_F)^{2(d-1)} \tag{17}$$

for a d-dimensional conductor, where W is the transverse dimension (the "width") of the conductor, and $k_F = 2\pi/\lambda_F$ is the Fermi wavenumber.

As a carrier proceeds through a particular channel in the sample, it suffers numerous elastic collisions, and the overall

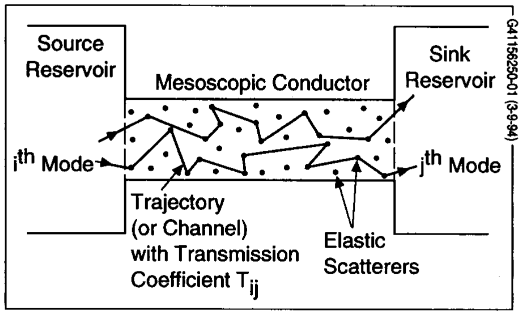

Fig. 1. Schematic representation of a two-terminal mesoscopic conductor, containing a random distribution of elastic scatterers and connected to a carrier reservoir at each end. The trajectory of a carrier starting in the i-th mode at the source reservoir and ending up in the j-th mode at the sink reservoir, and suffering numerous elastic collisions in between, defines a "channel." Since the electron must be viewed as a wave at this scale, the "trajectory" is entirely fictitious, and introduced for conceptual purposes only.

transmission coefficient of the channel will be a composite of the effect of each scattering. Since elastic scattering occurs at static lattice defects whose potential does not change with time, an electron in a given momentum state, incident at a given scatterer, will always be scattered into a specific momentum state. As a result, there is a well defined, time-invariant transmission through a given channel that can be described in terms of the transmission matrix $\mathbf{t}$ of the wave amplitude. The transmission coefficient for the carrier is denoted by T_{ij} for a channel between the i-th mode at the source end and the j-th mode at the sink end of the conductor.

The total current through the conductor is the sum of the currents transported through the individual "channels." Since the electrons obey Fermi-statistics, the various channels are equally populated, and contribute equally to the current transport. The conductance in this case is given by

$$G = (q^2/h)\mathrm{Tr}(\mathbf{t}\mathbf{t}^{\dagger}) = (q^2/h)\sum T_{ij} \tag{18}$$

where Tr represents the trace of the matrix, the dagger indicates the Hermitian conjugate, T_{ij} is the transmission coefficient, and the summation extends over all possible channels [25]. For a single channel case, (18) obviously reduces to (16).

IV. Conductance Fluctuations

The seemingly random variations in the conductance of a mesoscopic conductor with external fields, with time, and from sample to sample, can now be understood. Section IV-A describes a physical mechanism, based on quantum interference among electrons, as a possible explanation for the observed fluctuations. Conductance fluctuations arise from interference among electron waves that have traversed the conductor along different trajectories, thereby suffering different phase shifts during transmission through the conductor. As long as there are no phase-breaking scattering events, the carriers crossing the conductor along different trajectories will have definite phase differences among them, and will interefere with each other to yield the total current. It is clear that the total current, and hence the sample conductance, will be sensitive to any changes that influence the phase shift accumulated along a trajectory. Magnetic field, electric field, and rearrangement of the elastic scatterers in the lattice can each cause a change in phase shift, and hence a variation in conductance, that is sufficiently complex to be called "fluctuation."

To ascertain that the above quantum inerference mechanism is indeed responsible for the observed mesoscopic conductance fluctuations, it is necessary to examine it for the experimentally observed features of fluctuations. The two characteristic features of the rms value of conductance fluctuations are its universality, and its temperature dependence; they are deduced on an intuitive, order-of-magnitude basis in Sections IV-B and IV-C, respectively. A discussion of the more detailed and exact calculation of fluctuations is postponed until Section V.

A. Physical Mechanism of Fluctuations

The conductance formula in (18) shows that the sample conductance is governed entirely by the transmission coefficients T_{ij} for the set of all possible channels available to carriers for traversing the conductor. The summation in (18) is strongly influenced by the destructive and constructive interference among the electron wavefunctions passing through the various channels. The phase shift suffered by a wavefunction in propagation along a channel depends on the electron energy and the spatial arrangement of the lattice disorders that serve as the elastic scatterers in the lattice. A fluctuation in G can therefore arise from a change in either the electron energy or the exact locations of imperfections in the lattice. An energy change can be induced by applied fields, while a rearrangement of disorder can occur through repeated sample preparation or in time. Each of these possibilities is examined next.

1) Fluctuation with Fields: A change in the phase shift of carrier wavefunction can be brought about by applying an electrostatic potential or a magnetic field. An electrostatic potential, whether applied across the sample or through a gate electrode, directly changes the electrochemical potential of the carriers, and therefore the phase-shift, which is sensitive to the energy. This potential can be varied in order to alter the interference pattern between the electron waves. Let E_c be the width of the carrier energy levels due to the finite conductor length, given by

$$E_c = \hbar D/L^2. \tag{19}$$

Then a change in the applied voltage by E_c/q would alter the carrier energy by E_c, and is sufficient to cause the phase shifts along individual channels to become uncorrelated with their original value. As a result, the interference between each pair of channels will change and completely alter the conductance. If a magnetic field perpendicular to the direction of current flow is applied, the phase of the electron wavefunction will again shift in proportion to the integral of the magnetic vector potential $\mathbf{A}$. A magnetic field change of the order of ϕ_0/A will create a distinct interference pattern, where ϕ_0 is the quantum flux unit, and A the transverse area of the conductor. In effect, the application of a field creates a new conductor sample, having a different interference pattern among the channels.

2) Sample-to-Sample Fluctuations: Next, consider a change in the exact distribution of disorders within the lattice. Such changes would occur naturally among multiple samples prepared similarly, and account for the sample-to-sample fluctuations. Two conductors are macroscopically identical when they are characterized by the same macroscopic parameters, and any randomness in their structures is statistically the same (e.g., same density and type of defects), even though there are differences in detail (i.e., in the precise locations of the scattering centers). But macroscopic properties like conductance do not remain the same among nominally identical mesoscopic conductors, because they depend on the microscopic details. Even a small change (e.g., the relocation of a single scatterer) can potentially cause a large change in the conductance. To understand this assertion, consider a two-dimensional mesoscopic conductor, as shown in Fig. 1, in which the separation between the elastic scatterers is much larger than the Fermi wavelength. Then the electron traveling between two elastic scatterers can be treated as a plane wave, and the motion of the carrier can be viewed as a random walk, in which the carrier goes from one elastic scatterer to another, with a mean step size equal to the mean free path, and a speed equal to the Fermi velocity. If the conductor is strongly disordered, the carrier will visit a significant fraction of the scatterers in the sample, so that disturbing even a single scatterer will affect the phase of a significant fraction of the channels. As a result, the change in sample conductance can be just as large as that caused by disturbing a large fraction of the scatterers. The same argument applies in three-dimensional (bulk) conductors, but the fraction of channels affected can be expected to be smaller, so that the effect is less striking.

3) Temporal Fluctuations: Finally, consider the fluctuation of conductance with time. For a given sample, a change in the spatial distribution of scattering potentials in a conductor can also occur spontaneously in time due to the motion of the defects serving as the scattering centers. A lattice disorder (such as an impurity or a fault) can move through the lattice of a solid conductor in several different ways. The defect may migrate by diffusion along the grain boundary; it may exist in a meta-stable state which becomes thermally activated at high temperatures; or, it may move by quantum mechanical tunnelling through a localizing potential barrier, a process which can occur even at low temperature. The migration of a defect once again leads to a change in the transmissions of the channels, and hence to a variation in conductance.

B. Universality of the RMS Value of Fluctuations

The following explanation [26] of this observed characteristic in terms of Landauer's formula is not only simple and intuitive but is also the basis of a technique for the numerical simulation of the fluctuations, discussed in Section V-B. The rms value of conductance fluctuations for a conductor can be estimated by subdividing it into more elementary single-channel conductors. For a mesoscopic conductor, the conductance depends on the summation of transmittances over all of the $N_{\rm ch}$ channels in it, as given by (18). In the diffusive regime, the number of collisions is large, so that each channel consists of a path that involves scattering at a large number of centers; in such a system the transmissions of individual channels will not be vastly different from each other. However, the conductor cannot be treated as a parallel connection of $N_{\rm ch}$ independent conductors, each containing only one channel, because the transmissions in individual channels are highly correlated, being the result of scattering by the same set of lattice imperfections. Instead, it is equivalent (in overall conduction) to a conductor in which there is effectively a smaller number of channels that *are* independent of each other, each having a transmission coefficient equal to the typical value of the transmission through the actual channels. This effective number of channels $N_{\rm eff}$ can be estimated as follows.

In a conductor of length L and mean free path between collisions Λ, the average number of collisions suffered by a carrier in its transit through the conductor will be $(L/\Lambda)^2$. Since these collisions are the source of correlation, approximately $(L/\Lambda)^2$ different channels will give rise to one effectively independent channel. As a result

$$N_{\rm ch}/N_{\rm eff} \sim (L/\Lambda)^2. \tag{20}$$

Essentially the same result can also be arrived at by a more careful analysis, involving a consideration of the distribution of the logarithms of those eigenvalues of the transmission matrix in (18) that contribute to conduction. Next, the value of $N_{\rm ch}$ itself can be estimated in terms of the average conductance $\langle G\rangle$ with the help of (17), by recognizing $W^{(d-1)}$ as the transverse area of crosssection A of the conductor. Combining (15), (17), and (14) relates $N_{\rm ch}$ to the conductance as follows:

$$\langle G\rangle \sim (q^2/h)(\Lambda/L)\sqrt{N_{\rm ch}}. \tag{21}$$

Equations (20) and (21) together yield $N_{\rm eff}$ in the form

$$\langle G\rangle \sim (q^2/h)\sqrt{N_{\rm eff}}. \tag{22}$$

Consider next the elementary conductor with a single channel. The conductance of such a one-channel conductor, to be denoted by g, is expressed by (16). In this expression, the transmission coefficient of the channel can fluctuate between 0 and 1. Consequently,

$$\langle g\rangle \sim q^2/h \tag{23}$$

and

$$\Delta g_{\rm rms}/\langle g\rangle \sim 1 \tag{24}$$

are reasonable estimates; they describe the universality property for the rms value of the conductance of a one-channel conductor. Since the $N_{\rm eff}$ channels are independent, it follows from (4) that

$$\Delta G_{\rm rms}/\langle G\rangle \sim (1/\sqrt{N_{\rm eff}})\Delta g_{\rm rms}/\langle g\rangle. \tag{25}$$

Substituting the results of (22) and (24) into (25) leads to

$$\Delta G_{\rm rms} \sim q^2/h \tag{26}$$

and shows that the rms conductance fluctuation is a universal constant, independent of the $\langle G\rangle$ of the conductor as well as of its geometrical and material properties, whether related to the lattice or to the impurities therein.

The difference between the scaling of conductance fluctuations with sample size for a macroscopic and a mesoscopic conductor can now be understood. In a mesoscopic conductor, the transmission coefficient of a single channel does not scale with the conductor length L; the number of channels varies as the square of the transverse area; and the effective number of independent channels varies directly with the square of the transverse area as well as inversely with the square of the conductor length. By contrast, if a macroscopic conductor is subdivided into a set of independent conductors connected in parallel, then the conductance of each elementary conductor scales directly with area and inversely with L, while their number scales as the area. For the illustrative example of a three-dimensional cubic conductor mentioned in Section II-B, the relative fluctuation varies as $1/\sqrt{n}$ for macroscopic conductors, but as $1/n$ for mesoscopic conductors.

Breakdown of universality can occur for a variety of reasons. For example, due to magnetic, geometric, and contact effects, intersubband scattering, and spin-orbit interactions, calculation and verification of these effects is an active area of research.

C. Temperature Dependence of Fluctuations

The observed increase of the rms fluctuations in conductance with decreasing temperature is an unexpected result, and can be understood in terms of the temperature dependence of the phase coherence length L_ϕ of carriers. The mean free path is related to the mean free time by $\Lambda = \sqrt{D\tau}$, where τ is the mean free path and D the diffusivity of the carriers. As the temperature is lowered, τ_{in} increases, while tau_{el} remains unchanged. Moreover, at low temperatures, where the condition $\Lambda_{\text{el}} \ll \Lambda_{\text{in}}$ is met, $\tau_{\text{in}} \propto 1/T^m$ where $\boldsymbol{T}$ is the temperature, and m lies between 1 and 2.

As the temperature of a sample of mesoscopic conductor is increased, the phase coherence length L_ϕ of the carriers in the sample decreases rapidly. As a result, the size of the conductor, measured in the units of L_ϕ, becomes large. Such a sample can be viewed as being a composite of several independent subsamples, each of approximate dimensions L_ϕ, so that they can be treated independently. The wavefunction within one subsample is coherent, but is uncorrelated with that in another subsample. Since the subsamples are mutually independent, the fluctuations in the total conductance of the sample can be deduced in the manner of Section II-B. As the temperature increases, the number of subsamples increases, and the rms fluctuation in G decreases, in accordance with (4).

V. Calculation of Fluctuations

There are two principal methods for theoretically calculating the magnitude of conductance fluctuations, one analytical and the other numerical. Historically, the existence of a universal value for the relative fluctuations was first explained, and its magnitude quantitatively derived, by a perturbation method applied to the transport theory; the basic idea behind that analytical calculation is briefly outlined in Section V-A below. Numerical calculations of functuation magnitude are useful for parametric studies, and are illustrated in Section V-B.

A. Analytical Calculation by Perturbation Method

The analytical method of calculating the variance of conductance fluctuations relies on the so-called "ergodic hypothesis" due to Lee and Stone [16]. The hypothesis is motivated by the observation that the transmission coeffcient of electron wavefunction through a channel in the conductor can be changed in two totally different ways, one internal to the conductor and one external to it:

1) By rearrangement of lattice disorders or defects within the conductor sample.
2) By variation of the external parameters that influence the energy of carriers, namely the magnetic field B and the electrochemical potential E_F.

The ergodic hypothesis is based on the postulate that as the two external parameters are varied in the neighborhood of their nominal state (i.e., their "quiescent" values) as

$$B = B_0 + \Delta B; \quad E_F = E_{F0} + \Delta E_F \tag{27}$$

the transmission coefficients of the sample will be swept through the entire range of transmission coefficients attainable through the rearrangement of defects. If one accepts this premise, the fluctuations in conductance G that would occur by varying the exact locations of the defects (i.e., over an ensemble of conductors) can be observed from a single conductor, simply by varying the external parameters ΔB and ΔE_F. Formally, the ergodic hypothesis states that the variance in G values as a function of the external parameters B and E_F is equal to the variance in G values over an ensemble of conductors with varying defect configurations, but at a fixed value of B and E_F.

The ergodic hypothesis should be viewed only an an approximation and a convenient scheme for calculation. It is known to be invalid at high fields, and at low fields its applicability can be judged only through verifying the correctness of its consequences.

The utility of the ergodic hypothesis lies in the fact that the sensitivity of G to the magnetic field and the Fermi level can be explicitly calculated by the usual Green's function method of transport theory. As a result, the variance in G due to changes in B and E_F can be directly found; from ergodic hypothesis, this variance is equal to the sample-to-sample variance of conductance. The result of such a calculation [27] is as follows

$$\langle \Delta g^2 \rangle = \frac{6}{\pi^4}\left[\frac{q^2}{h}\right]^2 \times \sum_{m=1}^{\infty}\sum_{n=0}^{\infty} \frac{1}{[m^2 + (nL_x/L_y)^2 + (L_x/\pi\Lambda_{\text{in}})^2]^2} \tag{28}$$

for a mesoscopic conductor of length L_x (along the direction of current flow) and transverse dimension L_y, and an inelastic diffusion length $\Lambda_{\text{in}} = \sqrt{D\tau_{\text{in}}}$, where τ_{in} is the inelastic mean free time, and D is the effective diffusivity under the quiescent conditions.

B. Numerical Calculation

The conductance of a mesoscopic conductor, and the fluctuations in it, can also be calculated from carrier scattering matrix by numerical computation. The scattering matrix yields the transmission matrix **t**, and the conductance can be calculated from it using (18). A number of authors have carried out such computations. One representative example is briefly described here to illustrate the results obtained; the selected example also brings out some limitations of the universal conductance fluctuation theory due to localization effects in sample with longer lengths.

Tamura and Ando [28], [29] consider a two-dimensional conductor, with a sample length L along the direction of conduction, and a sample width W in the lateral direction. Each end of the conductor is assumed terminated in an ideal lead and a reservoir. The impurity atoms, which serve as the elastic scatterers, are assumed to have a δ-function like electrostatic potential, and to have a random distribution within the sample. The electrons are treated as mutually noninteracting, and the impurity strength is taken to be sufficiently small that the higher-order (Born) scattering from an impurity can also be neglected. Starting with the scattering matrix for a single impurity, the scattering matrix for the entire multi-impurity sample is synthesized, and then used to calculate the transmission coefficients. These in turn can be used to numerically compute the conductance G, and its moments over an ensemble of similar conductors. The computed results are shown in Fig. 2 for the case when no magnetic field is present.

The computed average conductance $\langle G \rangle$ of the two-dimensional sample depends on the sample dimensions in the manner shown in Fig. 2(a), where the sample length L is normalized with respect to the mean free path Λ, while the sample width is normalized with respect to the Fermi wavelength λ_F as in (15). The ratio of L to λ_F is kept constant, and equals 51.25. The ratio of the localization length ξ to Λ is found to be equal to the number of filled subbands, to within numerical accuracy. Several features of the results are noteworthy. Consider first the dependence of $\langle G \rangle$ on L for a fixed value of W. When L is very small, of the order of the mean free path Λ, the number of collisions suffered by the electrons in the sample is small, so that the carrier transport is nearly ballistic. In the limit of ballistic transport, the conductance simply has the quantized value, given by

$$G = N_{\mathrm{sb}}(2q^2/h) \tag{29}$$

and depends only on N_{sb}, the number of subbands that are filled (i.e., are below Fermi level). This number is approximately twice the sample width measured in units of Fermi wavelengths; its values are 5, 10, and 20 for the three cases considered in Fig. 2(a). As L increases, the sample begins to approach the diffusive regime where (18) applies, and the conductance decreases. When L becomes large compared to the coherence length ξ, the conductor can be considered as a series connection of L/ξ conductors, each of length ξ, and its total conductance decreases with increasing L in an exponential manner.

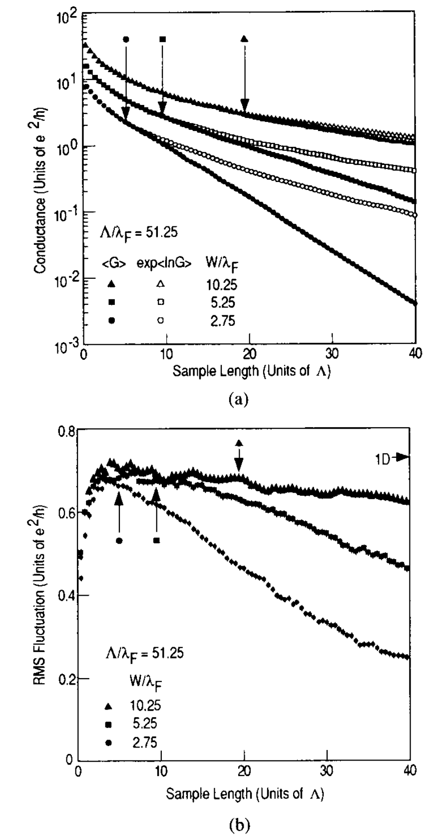

Fig. 2. Numerically computed conductance and rms conductance fluctuations in an idealized sample of conductor, as a function of the sample length L and for various values of sample width W. The length is normalized with respect to the carrier mean free path Λ, and the width with respect to the carrier Fermi wavelength λ_F. *(Reprinted from: H. Tamura and T. Ando, "Numerical and analytical studies of quantum transport in quantum wires," in Transport Phenomena in Mesoscopic Systems, H. Fukuyama and T. Ando, Eds., Proc. 14th Taniguchi Symp., Shima, Japan, Nov. 1991. Berlin: Springer-Verlag, 1992, pp. 117–128).*

Next consider the dependence of conductance on sample width W. An increase in W has two consequences. First, the number of occupied subbands increases linearly with W, so that the conductance G is increased by a similar factor. Second, the coherence length ξ becomes larger; as a result a larger value of L is necessary before the sample can be treated as multiple independent sections, and the dependence of $\langle G \rangle$ on L becomes exponential.

Fig. 2(a) brings out another issue. When the ensemble average of sample conductance is calculated, the samples with very large conductance values have an unduly large influence on the average value. An alternative is to define a logarithmic average as follows:

$$\langle G \rangle_{\log} = \exp[\langle \ln G \rangle]. \tag{30}$$

This average reduces the contribution of large G values, and is therefore lower. Fig. 2 shows that the logarithmic average deviates from $\langle G \rangle$ only when the sample length L is large compared to ξ, implying that the very large values of sample G occur only for long samples.

Consider next the rms value of fluctuations in conductance, shown in Fig. 2(b). For small values of L where the carrier transport is nearly ballistic, $\Delta G_{\rm rms}$ increases with increasing L, independent of W. When L is sufficiently large that the transport becomes diffusive, $\Delta G_{\rm rms}$ is independent of the sample dimensions and hence of $\langle G \rangle$. This is the regime where universal conductance fluctuations theory can be applied. When $L \gg \xi$, the magnitude of $\Delta G_{\rm rms}$ decreases with increasing L due to localization effects. When the sample width is small, the number of channels through the conductor is also small, and the localization effects set in for smaller value of L. As a result, for very small W, there is no constant $\Delta G_{\rm rms}$ region characteristic of UCF, lying in between the ballistic and localization regimes. The computed results show that, in the localization regime, $\Delta G_{\rm rms}$ is determained essentially by L/ξ, and is not sensitive to W.

VI. Experimental Observations

The volume of experimental work published on the subject is sizable; as a result, even a brief summary of each individual work is neither possible due to space limitations, nor necessary given the tutorial objectives of this paper. Instead, a collective summary of the entire literature is attempted in Section VI-A, and indicates the range of experimental methods used, and the generic conclusions drawn. However, the exact findings, and their associated caveats and limitations, can only be understood in the context of a specific experiment; these are illustrated by summarizing three experimental studies in Sections VI-B through VI-D, that are representative of the body of literature, and are selected to present a variety of experimental conditions, issues, and results.

A. Summary of Experimental Studies

1) Sample Material, Structure, and Preparation: The experimental observations of fluctuations in conductance, and other mesoscopic transport properties, have been carried out on a variety of samples, including thin films of Bi [30], Pt [31], Ag [32], Au [12], and Cu [33]; channels of Si MOS transistors [34], GaAs/AlGaAs semiconductor heterojunctions [35], and HgCdTe MISFET's [36]; and amorphous materials [37]. These include materials that have been evaporated, sputtered, deposited, and grown. The samples themselves have taken many different structural forms, including constrictions in thin film conductors, point contacts, air-bridges, and field-effect devices controlled by a gate. The dimensions of the conductor sample have been made ultrasmall, and controlled, in a number of ways including etching, selective growth, photolithographic patterning, electron beam lithography, electrostatic bounding (as in gate electrodes and depletion layers), and change in composition (as in heterostructures). And finally, samples tested have been quasi one-dimensional, two-dimensional, and three-dimensional.

2) Operating and Measurement Conditions: Mesoscopic fluctuations are observed not only in conductance but also in many other transport properties, including magnetoresistance, photovoltaic and thermoelectric coefficients, light transmittance through disordered media, and critical current in superconductors. The conductance fluctuations have been observed at various cryogenic temperatures below 10 K, down to milliKelvins. The studied variations (i.e., fluctuations) in conductance have been caused by magnetic field [38], applied voltage [39], multiple samples with similar preparation [35], and time [40]. The changes in sample conductance have been measured using dc as well as ac signals. Finally, the changes in the magnitude of fluctuations have been studied as a function of temperature [41], [42], thermal cycling [43], large magnetic fields [44], IR radiation [45], electrical shocks [46], and high-energy electron irradiation [47].

3) Conclusion: The observations, although different in detail, do not vary fundamentally with the conductors, or with their method of preparation. The general features of observations may be summarized as follows:

1) In the mesoscopic regime, the conductance fluctuations ($\Delta G \equiv G - \langle G \rangle$) have an rms value given by
$$\sqrt{\langle (\Delta G)^2 \rangle} \approx q^2/h \tag{31}$$
independent of the average value of the conductance $\langle G \rangle$, i.e., the fluctuations are scale invariant.
2) The magnitude of conductance fluctuations is independent of the sample size, the degree of disorder, or the microscopic details of the sample; hence the designation "universal."
3) As the temperature is lowered, the rms fluctuations in conductance increase in size, approaching the value in (31).
4) A small change in disorder (even the relocation of one lattice defect) can potentially cause a large change in the sample conductance.
5) The fluctuations have non-Gaussian distribution and other unexpected statistical properties.

B. Sample Size Dependence

The rms value of conductance fluctuations is expected to have a universal value, independent of the sample dimensions, only within the region of applicability of UCF theory. A measurement of the magnitude of fluctuations as a function of sample dimensions is therefore useful both for confirming this expectation, and for studying the transition to other regions. Two such transitions can be explored by changing sample dimensions. The longitudinal dimension (sample length L) determines whether the sample is in the ballistic or the diffusive regime; in the ballistic regime, there are no differential phase shifts of wave function among the possible channels through the sample due to an absence of scattering, therefore no quantum intereference and no conductance fluctuations are expected. The transverse dimension (sample width W) governs the number of channels through the sample, and therefore the number of wavefunctions among which quantum interference is possible.

One measurement of the sample-size dependence of fluctuations is reported by Ishibashi *et al.* [35]. Their conductor sample was the channel formed in an MBE-grown GaAs/AlGaAs heterojunction, maintained at a temperature of 1.2 K, having a mean free path of approximately 1 μm. Sample length was

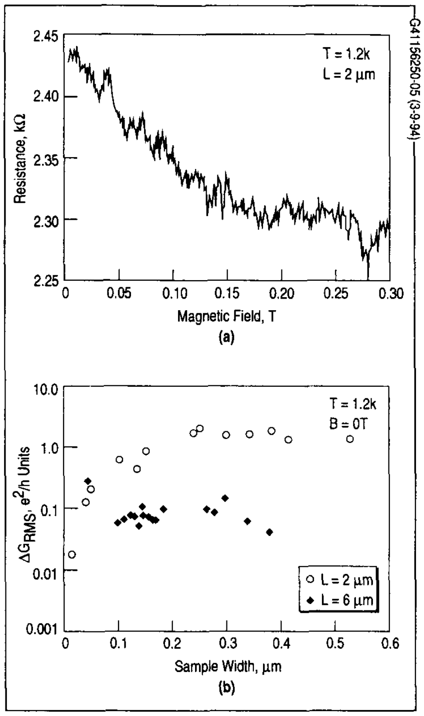

Fig. 3. Measured resistance and rms value of conductance fluctuations for channel at GaAs/AlGaAs heterojunction. (a) Sample resistance as a function of transverse magnetic field. (b) Conductance fluctuations as a function of sample dimensions. *(Reprinted from: K. Ishibashi, "Conductance fluctuations in GaAs/AlGaAs narrow wires in quasi-ballistic regime," Japan. J. Appl. Phys., vol. 31, Pt. 1, no. 12B, pp. 4504–4507, Dec. 1992).*

varied by fabricating samples of 2 and 6 μm lengths, while the sample width was governed by a negative voltage applied to a gate electrode; for gate voltages between -0.6 V (at which the electron gas is fully depleted), and -2.4 V (beyond which the carrier concentration is also simultaneously reduced), the sample width varied from 0.5 μm to a few percent of nm. Strong magnetic fields upto 8 T were used for measuring the magnetoresistance that is used for estimating phase coherence length L_ϕ; weaker fields below 1 T were used for causing fluctuations in sample conductance so as to measure its rms value. The variation of sample resistance with magnetic field is shown in Fig. 3(a), and the dependence of the ΔG_{rms} on sample width is shown in Fig. 3(b) for the two different sample lengths.

Their principal observations are as follows.

1) The fine structure of the field dependence of sample resistance shown in Fig. 3(a), is reproducible when carefully measured. However, it changes if the sample is subjected to thermal recycling or electric shock, which would be expected to modify the impurity configuration within the sample in microscopic detail.
2) For the sample of length 6 μm, which is in the diffusive regime since $L/\Lambda \approx 6$, the value of ΔG_{rms} is independent of the sample width, as would be expected from the UCF theory.
3) For the sample of 2 μm length, ΔG_{rms} increases with increasing sample width. This sample is in the quasiballistic transport regime, so that the correlations between the channels throught the sample should be weak due to the small number of scattering events. The effective number of channels N_{eff} approaches the actual number of channels N_{ch} as evident from (20), and N_{ch} is related to W by (17). This provides a qualitative explanation for the observation.
4) For larger sample widths where ΔG_{rms} becomes independent of W, its value for the 2 μm sample is about an order of magnitude larger than for the 6 μm sample. The phase coherence length L_ϕ, estimated from magnetoresistance measurements, is approximately 1 μm. Therefore the samples can be viewed as L/L_ϕ coherent subunits, connected together in series, with an averaging of fluctuations over a larger number of subunits for the longer sample. This accounts for a factor of 5 reduction in the values of ΔG_{rms} for the longer sample compared to the shorter one.

C. Time-Dependent Fluctuations

The measurement of conductance variations with time provides some additional information that is not obtainable through field-induced variation of conductance. First, it shows the natural time scale associated with the fluctuating processes, and thus allows their time constants to be determined; the same information in the frequency domain would be provided by the power spectral density of the fluctuations. Second, with suitable filtering, it permits the elementary events causing the fluctuations to be resolved in time and observed individually rather than collectively; this can help identify the specific features of the elementary events (e.g., their magnitude or rate) through which an influencing factor (such as temperature) affects the composite fluctuations.

One such measurement is reported by Meisenheimer and Giordano [32], whose mesoscopic conductor consisted of a sputtered thin silver film having a thickness around 10 nm, and with dimensions 1 μm $\times$ 1 μm. The estimated phase coherence length in these samples is approximately 0.2 μm at 79 mK. The principal phase-breaking mechanism is estimated to be electron-electron scattering, which has a characteristic length equal to the thermal diffusion length in (8).

The measured results are shown in Fig. 4. Fig. 4(a) shows the variation of sample resistance for different sample temperatures, after the faster fluctuations (with time constants of less than 5 min) have been filtered out. Two features of this result are noteworthy. First, the magnitude of fluctuations increases very rapidly as the temperature decreases. The temperature dependence of the rms value of conductance fluctuations, expressed in units of q^2/h, is shown in Fig. 4(b). Second, the arrows in the plots of Fig. 4(a) mark the ocassions where the resistance of the sample fluctuates away from, and then returns to, a nominal region in time intervals of the order of 1 hour. This is indicative of the motion of a scattering center between two locally stable states, resulting in two resistance states. The power spectral density of the fluctuations (in arbitrary units) is shown in Fig. 4(c), and displays a region of roughly $1/f$ dependence over approximately two decades of freqeuncy between 10^{-5} to 10^{-3} Hz.

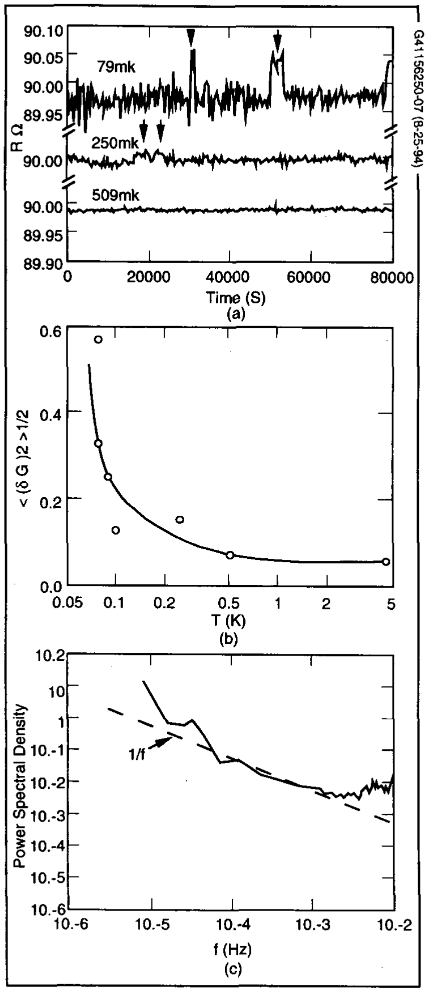

Fig. 4. Observed time-variation of the conductance of a thin-film silver conductor. (a) Resistance as a function of time for various sample temperatures. Arrows indicate reversible switching between two states. (b) RMS value of conductance as a function of sample temperature. (c) Power spectral density of fluctuations. Straight line denotes $1/f$ dependence. (*Reprinted from: T. L. Meisenheimer and N. Giordano, "Conductance fluctuations in thin silver films," Phys. Rev. B, vol. 39, no. 14, Issue I, pp. 9929–9936, May 1989*).

D. Impact of a Single Defect

Several of the observed features of conductance fluctuations are suggestive of the hypothesis that the reconfiguration of even a single defect in the conductor lattice leads to a measurable change in conductance and is therefore visible in the fluctuations. These features include the reversibility of fluctuation, the magnitude of discrete resistance change, and the magnetic field dependence of conductance measured over a range of fields many times the correlation field. A more definitive method of identifying the motion of a single defect as the cause of conductance fluctuation is through reducing the size of the conductor, and the separate measurement of the local intereference (LI) effect, which is an alternative mechanism of fluctuations.

The measurements reported by Ralls *et al.* [33] show the transition between the UCF and the LI fluctuations. Their conductor sample is essentially the thin film equivalent of a metal point contact, formed by deposition of evaporated copper on a via hole in silicon nitride, thereby creating a metal constriction of about 50 nm length and a 3–20 nm diameter, with wider electrodes at both ends. The conductor as grown is ballistic, having a mean free path of 180 nm, but the application of a voltage in the range of 100 to 500 mV causes Cu atoms to move by electromigration, causing the resistance to decrease, and creating a disorder in the constriction region between electrodes. The disorder so created is confined to a very small region near the constriction, leaving the electrode regions relatively ordered, with their high phase coherence length, so that they do not contribute to the observed fluctuations.

The sample conductance fluctuations are observed by applying a small dc bias voltage of approximately ± 10 mV. Measurements of conductance made at 4.2 K and zero magnetic field are shown in Fig. 5. The sample, as grown, is ballistic and displays no measurable conductance fluctuations. Electromigration creates a less-ordered material, and reduces the elastic mean free path, in the constriction region, and the measured sample conductance after the introduction of defects is shown in the lower plot in Fig. 5. Three features of this plot are noteworthy. First, the appearence of the fine structure in this plot indicates the presence of fluctuation effects due to the disordered material. Second, the fine structure does not change even in the presence of a magnetic field as large as 2.6 T, indicating that the correlation field for this sample is very large, and therefore the disordered region is very small. Finally, the rms value of conductance fluctuation is only a fraction of q^2/h, indicating that there is still a significant ballistic transport taking place throught the constriction.

Intermediate values of dc voltage bias, lying in between those used for conductance masurement (~ 10 mV) and those used to create disordered regions (~ 100 mV), can be used to reconfigure a single influencial scatterer. Such bias voltages applied in a transient manner cause the defect configuration to switch to a different state, and freeze the configuration until the bias is raised again. Conductance fluctuations measured in each of the two states show the presence of UCF, as indicated by the bias and magnetic field sensitivity of the fluctuations. The change in the rms value of the fluctuations depends on the applied bias voltage in a random manner, conistent with the expectation that a change in a single scatterer will influence the phase shifts of the interefering wavefunctions in a complex manner.

VII. Low Frequency and $1/f$ Noise

In the experimental work summarized in Section VI, the conductance fluctuations occur either as a function of a varying externally applied excitation, or spontaneously in time. A complementary method of observing the fluctuations is in the frequency domain, where they appear as low frequency conductance noise, and have a power spectral density that is approximately proportional to $1/f$, where f is the frequency.

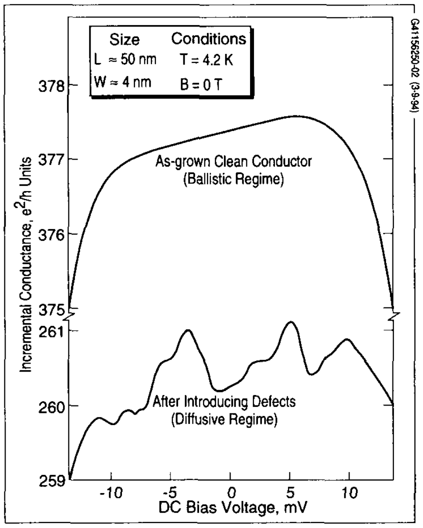

Fig. 5. Measured incremental conductance of a copper microbridge as a function of the dc bias voltage before and after the introduction of defects in the mesoscopic region. *(Reprinted from: K. S. Ralls, D. C. Ralph, and R. A. Buhrman, "Impact of a single defect on the conductance: Local interference and universal conductance fluctuations," Phys. Rev. B, vol. 47, no. 16, pp. 10509–10514, Apr. 1993).*

Since a variety of mechanisms are capable of producing low-frequency noise, the measured noise spectra can be expected to be a composite of the effects of various fluctuation mechanisms. The separation and identification of their individual contributions requires careful, painstaking work. Experiments indicate that for weakly disordered conductors in metallic regime at low temperatures, the fluctuations due to phase coherent effects are a dominant source of low-frequency noise.

A comprehensive treatment of the subject of low-frequency conductance noise would be lengthy, and will not be attempted here. Fortunately, good surveys including extensive bibliographies are available in the literature, both for the theoretical [48] and the experimental [49], [50] aspects of the subject. Several distinct mechanisms can give rise to conductance fluctuations of different types in ultra-small devices, of which the three principal types are the universal conductance fluctuations (UCF), the local interference fluctuations (LIF), and the strongly localized interference fluctuations (SLIF); other mechanisms may exist as well [51]. All of these types of fluctuations are based on the scattering of electron waves from impurities, but there are significant differences in their mechanisms, conditions of occurrence, and characteristics. The UCF have been discussed at length in this paper. The SLIF are caused by hopping conduction in semiconductors, where the conductance, as well as the relative fluctuations in conductance, depend exponentially on the size L of the conductor sample, measured in the units of localization length. Since a sample with UCF can, upon a suitable change of some parameters corresponding to a different regime, display low-frequency conductance fluctuations due to LIF, a brief description of this mechanisms is given in this section.

The UCF are observed in the "dirty metal" regime where $\Lambda_{\text{in}} \gg \Lambda_{\text{el}}$. There are two ways in which one can approach the "clean metal" regime where this inequality is reversed: either Λ_{in} can be decreased, or Λ_{el} can be increased. Λ_{in} can be decreased by raising the temperature, which increases the supply of phonons that cause inelastic scatterings. Λ_{el} can be increased by reducing the degree of disorder in the sample which reduces the number of elastic scatterers. In addition, if the elastic mean free path of the carriers is larger than the phase coherence length, the smallest independent part of the sample is no longer in the diffusive regime. The LIF are observed in such samples which have a higher purity, or are at a higher temperature, compared to those in which UCF are observed; and where the elastic and the inelastic mean free paths are of the same order, so that the phase coherence length L_ϕ is comparable to the mean free path Λ [52].

The conductance of a sample has been shown to be dependent on the transmission coefficients of the electron wave through the channels, which in turn depend on the scattering cross section of the scatterers. The influence of the migration of a defect on carrier transmission depends on the locations of other nearby defects. For example, if an isolated impurity atom in a lattice moves to a neighboring location, the lattice can be viewed as being merely displaced spatially, and the conductor remains unchanged. By contrast, if the migrating impurity atom was adjacent to a grain boundary, and is differently placed with respect to the boundary after migration, the net effect of both scatterers will be different, and the overall transmission will change. The motion of an impurity within the lattice can therefore cause a fluctuation in its scattering cross section. Local interference fluctuations are caused by a superposition of the cross section fluctuations. The resulting fluctuations in scattering rate, and hence in the conductance are observed as LIF.

REFERENCES

[1] R. Serra *et al.*, *Introduction to the Physics of Complex Systems: The Mesoscopic Approach to Fluctuations, Nonlinearity, and Self-Organization*. New York: Pergamon, 1986.
[2] M. A. Reed and W. P. Kirk, Eds., *Nanostructure Physics and Fabrication*. Boston: Academic, 1989.
[3] B. L. Altshuler, P. A. Lee and R. A. Webb, Eds., *Mesoscopic Phenomena in Solids: Modern Problems in Condensed Matter Sciences*, vol. 30. Amsterdam, North-Holland, 1991.
[4] H. A. Cerdeira, F. G. Lopen, and U. Weiss, Eds., *Proc. Adriatic Research Conf. on Quantum Fluctuations in Mesoscopic and Macroscopic Systems*. Singapore: World Scientific, 1991.
[5] D. K. Ferry, J. R. Barker, and C. Jacoboni, Eds., *Granular Electronics*, Proc. NATO Advanced Study Institute on Physics of Granular Nanoelectronics, Il Ciocco, Italy, July 1990. New York: Plenum, 1991.
[6] B. Kramer, *NATO Advanced Study Institute on Quantum Coherence in Mesoscopic Systems*. New York: Plenum, 1991.
[7] M. A. Reed, Ed., *Nanostructure Systems*. New York: Academic, 1991.
[8] J. H. Davies and A. R. Long, Eds., *Physics of Nanostructures*, Proc. 38th Scottish Univ. Summer School. Bristol, UK: Institute of Physics, 1992.
[9] H. Fukuyama and T. Ando, Eds., *Transport Phenomena in Mesoscopic Systems, Proc. 14th Taniguchi Symp.*, Shima, Japan, Nov. 1991. Berlin: Springer-Verlag, 1992.
[10] W. P. Kirk and M. A. Reed, *Nanostructures and Mesoscopic Systems: Proc. Int. Symp.*, Santa Fe, NM, May 1991. New York: Academic, 1992.
[11] T. Musha, S. Sato, and M. Yamamoto, *Noise in Physical Systems and 1/f Fluctuations*. Amsterdam: IOS Press, 1992.
[12] C. P. Umbach, S. Washburn, R. B. Laibowitz, and R. A. Webb, "Magnetoresistance of small, quasi-one-dimensional normal-metal rings and lines," *Phys. Rev. B*, vol. 30, no. 7, pp. 4048–4051, Oct. 1984.

[13] S. Washburn, C. P. Umbach, R. B. Laibowitz, and R. A. Webb, "Temperature dependence of the normal-metal Aharonov-Bohm effect," *Phys. Rev. B*, vol. 32, no. 7, pp. 4789–4792, Oct. 1985.

[14] A. D. Stone, "Magnetoresistance fluctuations in mesoscopic wires and rings," *Phys. Rev. Lett.*, vol. 54, no. 25, pp. 2692–2695, June 1985.

[15] B. L. Al'tshuler, "Fluctuations in the extrinsic conductivity of disordered conductors," *JETP Lett.*, vol. 41, no. 12, pp. 648–651, June 1985.

[16] P. A. Lee and A. D. Stone, "Universal conductance fluctuations in metals," *Phys. Rev. Lett.*, vol. 55, no. 15, pp. 1622–1625, Oct. 1985.

[17] B. L. Al'tshuler and D. E. Khmel'nitskii, "Fluctuation properties of small conductors," *JETP Lett.*, vol. 42, no. 7, pp. 359–362, Oct. 1985.

[18] W. J. Skocpol *et al.*, "Universal conductance fluctuations in silicon inversion-layer nanostructures," *Phys. Rev. Lett.*, vol. 56, no. 26, pp. 2865–2868, June 1986.

[19] S. Feng, P. A. Lee, and A. D. Stone, "Sensitivity of the conductance of a disordered metal to the motion of a single atom: implications for $1/f$ noise," *Phys. Rev. Lett.*, vol. 56, no. 18, pp. 1960–1963, May 1986.

[20] M. S. Gupta, "Mesoscopic conductors at low temperatures," *IEEE Circuits & Devices*, to be published.

[21] T. Ikoma and T. Hiramoto, "Weak localization and phase breaking mechanisms of electron waves in quasi one dimensional wires," in *Granular Electronics*, D. K. Ferry *et al.*, Eds. New York: Plenum, 1991, pp. 255–275.

[22] A. D. Stone and Y. Imry, "Periodicity of the Aharonov-Bohm effect in normal-metal rings," *Phys. Rev. Lett.*, vol. 56, no. 2, pp. 189–192, Jan. 1986.

[23] R. Landauer, "Electrical resistance of disordered one-dimensional lattices," *Philosophical Mag.*, vol. 21, no. 172, pp. 863–867, Apr. 1970.

[24] D. S. Fisher and P. A. Lee, "Relation between conductivity and transmission matrix," *Phys. Rev. B*, vol. 23, no. 12, pp. 6851–6854, June 1981.

[25] M. Buttiker, Y. Imry, R. Landauer, and S. Pinhas, "Generalized many channel conductance formula with application to small rings," *Phys. Rev. B*, vol. 31, no. 10, pp. 6207–6215, May 1985.

[26] Y. Imry, "Active transmission channels and universal conductance fluctuations," *Europhysics Lett.*, vol. 1, no. 5, pp. 249–256, Mar. 1986.

[27] A. D. Stone, "Theory of coherent quantum transport," in *Physics of Nanostructures*, J. H. Davies and A. R. Long, Eds., *Proc. 38th Scottish Universities Summer School.* Bristol, UK: Institute of Physics, 1992, pp. 65–100.

[28] H. Tamura and T. Ando, "Numerical and analytical studies of quantum transport in quantum wires," in *Transport Phenomena in Mesoscopic Systems*, H. Fukuyama and T. Ando, Eds., *Proc. 14th Taniguchi Symp.*, Shima, Japan, Nov. 1991. Berlin: Springer-Verlag, 1992, pp. 117–128.

[29] H. Tamura and T. Ando, "Conductance fluctuations in quantum wires," *Phys. Rev. B*, vol. 44, no. 4, pp. 1792–1800, July 1991.

[30] N. O. Birge, B. Golding, and W. H. Haemmerle, "Conductance fluctuations and $1/f$ noise in Bi," *Phys. Rev. B*, vol. 42, no. 5, pp. 2735–2743, Aug. 1990.

[31] T. L. Meisesnheimer, D. E. Beutler, and N. Giordano, "Conductance fluctuations in thin metal films," *Japan. J. Appl. Phys.*, vol. 26, pp. 695–696, 1987.

[32] T. L. Meisenheimer and N. Giordano, "Conductance fluctuations in thin silver films," *Phys. Rev. B*, vol. 39, no. 14, Issue I, pp. 9929–9936, May 1989.

[33] K. S. Ralls, D. C. Ralph, and R. A. Buhrman, "Impact of a single defect on the conductance: local interference and universal conductance fluctuations," *Phys. Rev. B*, vol. 47, no. 16, pp. 10509–10514, Apr. 1993.

[34] D. A. Slimmer, J. C. Licini, S. D. van Campen and R. F. Kwasnick, "Quantum conductance fluctuations in self-aligned, pinch-narrowed silicon metal-oxide-semiconductor field-effect transistors," *J. Appl. Phys.*, vol. 72, no. 3, pp. 1183–1185, Aug. 1992.

[35] K. Ishibashi *et al.*, "Conductance fluctuations in GaAs/AlGaAs narrow wires in quasi-ballistic regime," *Japan. J. Appl. Phys.*, vol. 31, Pt. 1, no. 12B, pp. 4504–4507, Dec. 1992.

[36] E. C. Palm and W. P. Kirk, "Quantum interference effects in mesoscopic narrow-gap systems with strong spin-orbit scattering," *20th Int. Conf. Physics of Semicond.*, Thessaloniki, Greece, Aug. 1990. Singapore: World Scientific, 1990, pp: 2243–2346.

[37] C. E. Parman, N. E. Israeloff, and J. Kakalios, "Nongaussian $1/f$ noise and conductance fluctuations in hydrogenated amorphous silicon," *Amorphous Silicon Technology—1992*, M. J. Thomson *et al.*, Eds. Pittsburgh: Material Research Soc., 1992, pp. 741–746.

[38] N. M. Zimmerman, B. Golding, and W. H, Haemmerle, "Magnetic field tuned enrgy of a simple two-level system in a mesoscopic metal," *Phys. Rev. Lett.*, vol. 67, no. 10, pp. 1322–1325, Sept. 1991.

[39] D. Mailly, M. Sanquer, J.-L. Pritchard, and P. Pari, "Reduction of quantum noise in a GaAlAs/GaAs heterojunction by a magnetic field: an orthogonal-to-unitary Wigner statistics transition," *Europhys. Lett.*, vol. 8, no. 5, pp. 471–476, Mar. 1989.

[40] Y. Ochiai et al., "Time dependent resistance changes in a narrow AlGaAs-GaAs wire," *Superlattices and Microstructures*, vol. 6, no. 3, pp. 337–340, 1989.

[41] M. W. Keller et al., "Effect of spin-orbit scattering on the mesoscopic conductance fluctuation amplitude in GaAs/AlGaAs heterojunctions," in *Granular Electronics*, D. K. Ferry *et al.*, Eds. New York: Plenum, 1991, pp. 511–514.

[42] J. R. Gao *et al.*, "Temperature dependence of universal conductance fluctuations in narrow mesoscopic Si inversion layers," *Phys. Rev. B*, vol. 40, no. 17, pp. 11676–11682, Dec. 1989.

[43] M. Davison et al., "Mesoscopic charge mapping by conductance fluctuations," *Physica B*, vol. 165–166, no. 2, pp. 865–866, Aug. 1990.

[44] A. K. Geim, P. C. Main, L. Eaves, and P. H. Beton, "Quantum transport in diffusive microstructures in high magnetic field," *Superlatices and Microstructures*, vol. 13, no. 1, pp. 11–20, 1993.

[45] S. J. Klepper *et al.*, "The effect of elastic scatterers on mesoscopic conductance fluctuatons in GaAs/AlGaAs heterojunctions," in *Granular Electronics*, D. K. Ferry *et al.*, Eds. New York: Plenum, 1991, pp. 519–521.

[46] S. Yin, N. J. Long and G. Bergmann, "Electrically induced jumps in the magnetofingerprints of nanostructures," *Solid State Commun.*, vol. 85, no. 1, pp. 51–55, Jan. 1993.

[47] G. I. Andersson, M. O. Andersson, and O. Engstrom, "Discrete conductance fluctuation in silicon emitter junctions due to defect clustering and evidence for structural changes by high energy electron irradiation and annealing," *J. Appl. Phys.*, vol. 72, no. 7, pp. 2680–2691, Oct. 1992.

[48] S. Feng, "Conductance fluctuations and $1/f$ noise magnitudes in small disordered structures: Theory," in *Mesoscopic Phenomena in Solids, Modern Problems in Condensed Matter Sciences*, vol. 30, B. L. Altshuler, P. A. Lee, and R. A. Webb, Eds. Amsterdam: North-Holland, 1991, pp. 107–129.

[49] N. Giordano, "Defect motion and low frequency noise in disordered metals," *Revs. Solid State Science*, vol. 3, no. 1, pp. 27–69, Mar. 1989.

[50] N. Giordano, "Conductance fluctuations and low-frequency noise in small disordered systems: Experiments," in *Mesoscopic Phenomena in Solids, Modern Problems in Condensed Matter Sciences*, vol. 30, B. L. Altshuler, P. A. Lee, and R. A. Webb, Eds. Amsterdam: North-Holland, 1991, pp. 131–171.

[51] E. C. Palm and W. P. Kirk, "Voltage fluctuations in a mesoscopic conductor," in *Nanostructures and Mesoscopic Systems*, W. P. Kirk and M. A. Reed, Eds., *Proc. Int. Symp.*, Santa Fe, NM, May 1991. New York: Academic, 1992, pp. 433–442.

[52] J. Pelz and J. Clarke, "Quantitative 'local-interference' model for $1/f$ noise in metal films," *Phys. Rev. B*, vol. 36, no. 8, pp. 4479–4482, Sept. 1987.

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